Thursday, April 19 $**$ *Worksheet 21 - Parametrizations and Integrals*

1. Consider the ellipsoid with implicit equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$

   (a) Parametrize this ellipsoid.

   (b) Set up, but do not evaluate, a double integral that computes its surface area.

2. Let

$$\mathbf{r}(u,v)=\langle(2+\cos u)\cos v,(2+\cos u)\sin v,\sin u\rangle,$$

   where $0\leq u\leq 2\pi$ and $0\leq v\leq 2\pi$.

   (a) Sketch the surface parametrized by this function.

   (b) Compute its surface area.

3. Consider the surface integral

$$\iint_\Sigma z\,dS$$

   where Σ is the surface with sides S_1 given by the cylinder $x^2+y^2=1$, S_2 given by the unit disk in the xy-plane, and S_3 given by the plane $z=x+1$. Evaluate this integral as follows:

   (a) Parametrize S_1 using (θ,z) coordinates.

   (b) Evaluate the integral over the surface S_2 without parametrizing.

   (c) Parametrize S_3 in Cartesian coordinates and evaluate the resulting integral using polar coordinates.

4. Let C be the circle in the plane with equation $x^2+y^2-2x=0$.

   (a) Parametrize C as follows. For each choice of a slope t, consider the line L_t whose equation is $y=tx$. Then the intersection $L_t\cap C$ of L_t and C contains two points, one of which is $(0,0)$. Find the other point of intersection, and call its $x-$ and $y-$coordinates $x(t)$ and $y(t)$. Compute a formula for $\mathbf{r}(t)=\langle x(t),y(t)\rangle$. Check your answer with your TA.

   (b) Suppose that $t=\dfrac{p}{q}$ is a rational number. Show that $x(p/q)$ and $y(p/q)$ are also rational numbers. Explain how, by clearing denominators in $x(p/q)-1$ and $y(p/q)$, you can find a a triple of integers U, V, and W for which $U^2+V^2=W^2$.

   (c) Compute $\int_C \frac{1}{2}\langle -y,x\rangle\cdot d\mathbf{r}$ using your parametrization above.