

Tuesday, January 24 ** Worksheet 3 - Parametric curves defined using vector arithmetic.

1. Let $f(x) = x^2 + x - 2$.

- (a) Graph the equation $y = f(x)$. (By hand, then check with a calculator if you want.)
- (b) Find the slope and equation of the tangent line to $y = f(x)$ when $x = 2$. Draw the tangent line on your picture.
- (c) Draw a vector in \mathbb{R}^2 that describes the direction of the line. Find a numeric representation of your vector.

2. Consider the curve given parametrically by

$$\begin{cases} x(t) = t \\ y(t) = t^2 + t - 2 \end{cases} \quad \text{for } 0 \leq t < 4.$$

- (a) Sketch the curve. How does this graph differ from your graph in Problem 1(a)?
 - (b) Consider the vectors formed by the pair $(x(t), y(t))$. Anchoring the vectors at the origin, sketch on your graph the vectors at time $t = 0, 1, 2, 3$.
 - (c) Now consider the vectors formed by $(x'(t), y'(t))$. Evaluate $(x'(t), y'(t))$ at time $t = 2$, what does the vector represent? Hint: Graph it on the curve at the point $(x(2), y(2))$.
 - (d) Imagine that the curve is the path of a moving particle. What is the speed of the particle when $t = 2$?
3. (a) Sketch the vector emanating from the origin ending at the point $(-5, 2)$ in \mathbb{R}^2 .
- (b) On the same graph and using the "head-to-tail" geometric addition method, draw the vector $(-5, 2) + (3, -1)$.
- (c) Do the same for $(-5, 2) + 2(3, -1)$.
- (d) Do the same for $(-5, 2) + (-1)(3, -1)$.
- (e) If we allow the scalar multiplying the vector $(3, -1)$ to vary, what geometric object is described by the parametric equation $(-5, 2) + t(3, -1)$ for all t ?

4. Consider the set of points in \mathbb{R}^3 defined by the parametric equation

$$\mathbf{l}(t) = (-5 + 2t, 2 + 3t, 1 - t) \quad \text{for all } t.$$

- (a) Using the properties of vector arithmetic, factor $\mathbf{l}(t)$ into the form $\mathbf{p} + t\mathbf{v}$ where \mathbf{p} and \mathbf{v} are vectors in \mathbb{R}^3 .
 - (b) Using the factored form (and your technique from Problem 3) sketch this object in \mathbb{R}^3 . Geometrically, what does this parametric equation describe?
 - (c) Why is the vector \mathbf{v} in your factored form referred to as the *direction vector*?
5. Let $\mathbf{a} = (-\sqrt{3}, 0, -1, 0)$ and $\mathbf{b} = (1, 1, 0, 1)$ be vectors in \mathbb{R}^4 .
- (a) Find the distance between the points $(-\sqrt{3}, 0, -1, 0)$ and $(1, 1, 0, 1)$.
 - (b) Find the angle between \mathbf{a} and \mathbf{b} .