**Math 285 - Intro Differential Equations**
**Spring 2011, sections G1 and X1**
**Practice Midterm 2, Monday April 11**

**Name:** ________________________________________

**Section (circle):** X1 at 12pm G1 at 3pm

**This is a practice exam. The real exam will consist of at most 4 problems.**

**In the real exam, no calculators, electronic devices, books, or notes may be used.**

**Show your work. No credit for answers without justification.**

**You may omit units for ease of reading, except in final answers which require units.**

**Good luck!**

1. ________/15
2. ________/15
3. ________/10
4. ________/10
5. ________/10
6. ________/10

Total: ________/70

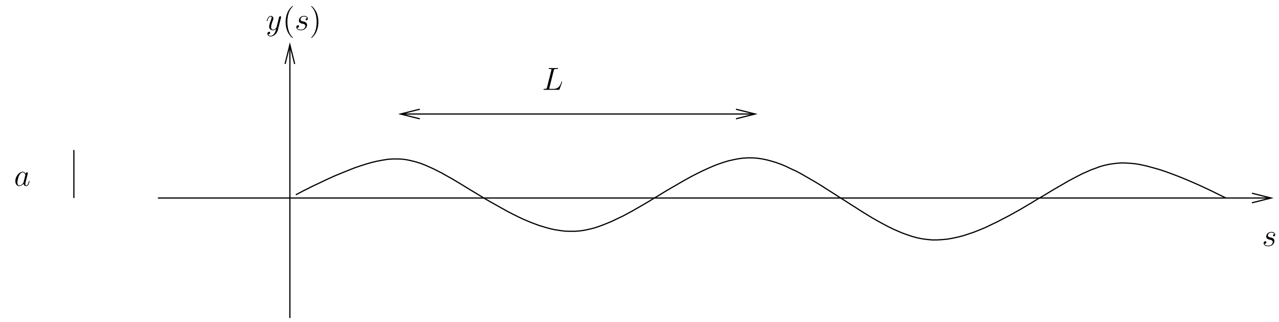


Figure 1: Bumpy road.

**Problem 1a. (12 pts)** A car is moving at constant speed $v$ on a road which becomes bumpy. The equation of the surface is $y(s) = a\sin(\frac{2\pi s}{L}) = 0.1\sin(\frac{2\pi s}{6})$ where $a = 0.1$ m is the amplitude of the bumps and $L = 6$ m is their wavelength (figure 1).

Assume the car behaves like a mass of $1,000$ kg on a spring with spring constant $100,000$ N/m, without shock absorber. We want to know how the bumpy road will affect the car. Let $x(t)$ be the upward displacement of the car, relative to its static equilibrium position – i.e. the equilibrium position once the spring has been compressed by the weight of the car.

Find $x(t)$, assuming the car was always at its vertical equilibrium position before reaching the first bump (at time $t = 0$). Assume a generic value of the speed $v$. Hint: The spring is stretched by $x - y$ and the spring force is the only force to take into account.

**b. (3 pts)** At what speed $v$ do resonance vibrations occur?

**Problem 2a. (9 pts)** A mass-spring-dashpot system with mass 1 kg, spring constant 4.5 N/m, and damping coefficient 1 N/(m/s) is subject to an external force $F(t) = 10\cos\omega t$. Find the steady periodic solution $x_{sp}(t)$.

**b. (6 pts)** Determine whether practical resonance can occur. If so, at what frequency (in Hertz), and what is the (maximal) amplitude of the steady periodic solution then?

**Problem 3a. (2 pts)** Check that $y_1(x) = x + 1$ and $y_2(x) = e^x$ are solutions of the equation:

$$xy'' - (x+1)y' + y = 0.$$

**b. (7 pts)** Find a particular solution $y_p(x)$ of the equation:

$$xy'' - (x+1)y' + y = x^2.$$

**c. (1 pt)** Find the general solution of the equation in (b).

**Problem 4a. (8 pts)** Find a periodic solution of the equation $x'' + 5x = f(t)$ where $f(t) = t$ on the interval $-\pi < t < \pi$ and $f$ is $2\pi$-periodic. You can express the answer in exact form or as a Fourier series – or both, for practice.

**b. (2 pts)** Find **all** possible initial conditions $x(0)$, $x'(0)$ such that the solution to the above equation is periodic. Explain.

**Problem 5. (10 pts)** A piece with mass $m$ inside a mechanical device is moving along an axis and is subject to a force $f(t)$, where $f(t) = 23\,|t|$ (in Newtons) for $-2 \le t \le 2$ and $f$ is periodic with period 4 s. The piece is also constrained by what is effectively a spring with spring constant 10 N/m. Friction is negligeable.

Find **all values** of the mass $m$ such that the piece could enter into resonance.

**Problem 6. (10 pts)** Find all eigenvalues and corresponding eigenfunctions $y(x)$ of the problem:

$$y'' + \lambda y = 0$$
$$y'(-1) = 0, \; y'(1) = 0.$$