Math 416 - Abstract Linear Algebra
Fall 2011, section E1
Additional problems

Section 2.6

A6.1. Prove the coordinate-free version of theorem 6.1, which we state here.

Theorem (General solution of a linear equation). Let V, W be vector spaces and $T \colon V \to W$ a linear transformation. Let $b \in W$, and let $x_p \in V$ be a particular solution of the equation $Tx = b$ (if there is one). Then the general solution of the equation $Tx = b$ is

$$x_p + \ker T = \{x_p + x_h \mid x_h \in \ker T\}.$$

Note that $\ker T = \{x \in V \mid Tx = 0\}$ is the general solution of the associated **homogeneous** equation $Tx = 0$.

Remark: No need to assume that V and W are finite dimensional. This makes the theorem useful in various contexts, such as solving linear differential equations.