Math 416 - Abstract Linear Algebra
Fall 2011, section E1
Additional problems

Section 2.8

A8.1. Let L be the line through the origin in $\mathbb{R}^2$ at an angle $\theta = \frac{\pi}{6} = 30^\circ$ from the x-axis (figure 1). In other words: $L = \text{Span}\{\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}\} = \text{Span}\{\begin{bmatrix}\frac{\sqrt{3}}{2} \\ \frac{1}{2}\end{bmatrix}\}$.

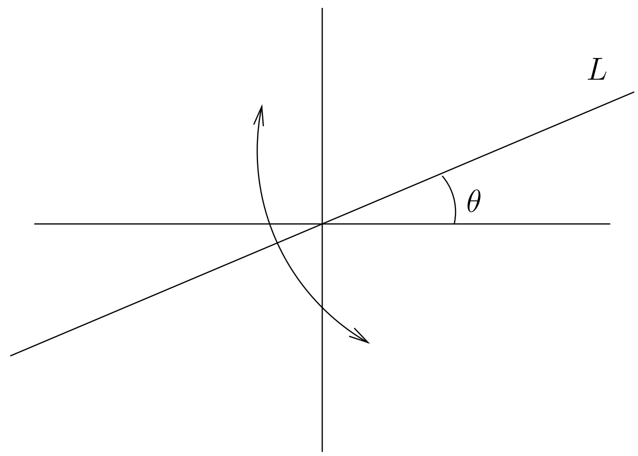

Figure 1: Reflection across a line.

Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection across the line L.

a. Let $\{v_1, v_2\}$ be the basis of $\mathbb{R}^2$ obtained by rotating the standard basis by an angle θ, i.e.

$$v_1 = \begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{3}}{2} \\ \frac{1}{2}\end{bmatrix}$$

$$v_2 = \begin{bmatrix}-\sin\theta \\ \cos\theta\end{bmatrix} = \begin{bmatrix}-\frac{1}{2} \\ \frac{\sqrt{3}}{2}\end{bmatrix}.$$

Note that v_1 lies on the line L and v_2 is perpendicular to it.

Find the matrix representing T with respect to the basis $\{v_1, v_2\}$.

b. Find the **standard** matrix representation of T using part (a) and the change of coordinates formula.

c. Confirm your answer in part (b) by computing $T(e_1), T(e_2)$ directly i.e. geometrically.

Remark: The matrices you obtained in parts (a) and (b) should have the same trace.