Math 416 - Abstract Linear Algebra
Fall 2011, section E1
Additional problems

Section 6.5

A6.5.1. Show that planar rotations in orthogonal planes commute with each other.

In other words, let P, P' be planes (through the origin) in $\mathbb{R}^n$ that are orthogonal to each other ($P \perp P'$). Let $R, R' \colon \mathbb{R}^n \to \mathbb{R}^n$ be planar rotations in the planes P, P' respectively. Show the equality $RR' = R'R$.

A6.5.2. Consider the vectors

$$v = \begin{bmatrix} 1+i \\ 1+i \\ 1-i \\ 1-i \end{bmatrix}, \; w = \begin{bmatrix} 1+i \\ -1-i \\ 1-i \\ -1+i \end{bmatrix}.$$

Let A be the 4×4 matrix with eigenvectors $v, \overline{v}, w, \overline{w}$ corresponding respectively to the eigenvalues $e^{i\theta_1}, e^{-i\theta_1}, e^{i\theta_2}, e^{-i\theta_2}$, for some $\theta_1, \theta_2 \in \mathbb{R}$.

One can show that A is orthogonal. [Not requested on the homework, although it's a fun exercise.]

Express $A \colon \mathbb{R}^4 \to \mathbb{R}^4$ as a product of two commuting planar rotations. Feel free to leave the answers as products of matrices (and possibly inverses).