Math 527 - Homotopy Theory
Spring 2013
Homework 13, Lecture 4/15

Recall one of the theorems stated and proved in the lecture.

Theorem. *Any path-connected topological abelian monoid is weakly equivalent to a product of Eilenberg-MacLane spaces.*

Problem 1. Show that any topological abelian *group* is weakly equivalent to a product of Eilenberg-MacLane spaces. (It need not be path-connected.)

Problem 2. Let X be a pointed CW complex, let $n \geq 0$, and let A be an abelian group. Show that there is a weak equivalence

$$\mathrm{Map}_* \left(X, K(A,n)\right) \simeq \prod_{k=0}^{n} K\left(\widetilde{H}^{n-k}(X;A), k\right).$$

Exercise (for fun, not to be turned in). Consider the circle with a disjoint basepoint $S^1_+ = S^1 \amalg \{e\}$ where e serves as basepoint. Show that the infinite symmetric product $\mathrm{Sym}(S^1_+)$ is *not* weakly equivalent to a product of Eilenberg-MacLane spaces.

This shows that the assumption of path-connectedness in the theorem cannot be dropped in general.