Math 527 - Homotopy Theory
Spring 2013
Homework 14, Lecture 4/22

Problem 1. Let $n \geq 2$. For any $f \in \pi_{2n-1}(S^n)$, denote its Hopf invariant by $H(f) \in \mathbb{Z}$.

a. Show that if n is odd, then $H(f) = 0$ holds for all $f \in \pi_{2n-1}(S^n)$.

b. Show that if n is not a power of 2, then there is no $f \in \pi_{2n-1}(S^n)$ satisfying $H(f) = 1$.

Hint: Use Steenrod squares.

c. Let $\eta\colon S^3 \to S^2$ and $\nu\colon S^7 \to S^4$ denote the Hopf bundles. Show that these two maps satisfy $H(\eta) = \pm 1$ and $H(\nu) = \pm 1$.

Note: Feel free to use known facts about quaternionic projective space $\mathbb{H}P^n$ and its cohomology, c.f. Hatcher Theorem 3.12 and the two pages that follow.