Math 527 - Homotopy Theory
Spring 2013
Homework 14, Lecture 4/24

Problem 2. Let n be a positive *even* integer. Let $\iota \in \pi_n(S^n)$ denote the class of the identity map, and consider the Whitehead product $[\iota, \iota] \in \pi_{2n-1}(S^n)$. Show that its Hopf invariant $H([\iota, \iota])$ is equal to 2.

Hint: Consider the map of cofiber sequences

$$\begin{array}{ccccc} S^{2n-1} & \xrightarrow{W} & S^n \vee S^n & \longrightarrow & S^n \times S^n \\ {\scriptstyle \mathrm{id}}\downarrow & & \downarrow{\scriptstyle \nabla} & & \downarrow \\ S^{2n-1} & \xrightarrow[[\iota,\iota]]{} & S^n & \longrightarrow & C([\iota, \iota]) \end{array}$$

where ∇ denotes the fold map.