Math 527 - Homotopy Theory
Spring 2013
Homework 1, Lecture 1/18

Problem 4. For pointed spaces, show that the smash product distributes over the wedge. More precisely, there is a natural isomorphism

$$X \wedge (Y \vee Z) \cong (X \wedge Y) \vee (X \wedge Z).$$

of pointed spaces. Don't forget to argue that the isomorphism is natural.

Edit 1/23/2013: A previous version of the homework contained a warning, but I was overly cautious. The map

$$X \times (Y \amalg Z) \to X \times (Y \vee Z)$$

is in fact a quotient map for *any* spaces X, Y, Z and basepoints $y_0 \in Y$ and $z_0 \in Z$. Proving this is a fun exercise in point-set topology.

It is when dealing with an arbitrary (infinite) wedge

$$X \times (\coprod_i Y_i) \to X \times (\bigvee_i Y_i)$$

that mild assumptions are needed.

In the upcoming lectures, we will learn how to circumvent point-set difficulties when doing homotopy theory.