Problem 3. Let X be the topologist's sine curve:

$$X = \{0\} \times [-1, 1] \cup \{(x, \sin \frac{1}{x}) \mid 0 < x \leq 1\}.$$

Consider the map $f \colon S^0 \to X$ which picks out the points $(0, 1)$ and $(1, \sin 1)$. Show that this map f is a weak homotopy equivalence but not a homotopy equivalence.