Problem 4. (May § 9.4 Lemma) Let $n \geq 2$ and consider the n-dimensional real projective space $\mathbb{R}P^n$. Show that the following holds: $\pi_1(\mathbb{R}P^n) \simeq \mathbb{Z}/2$ and $\pi_k(\mathbb{R}P^n) \simeq \pi_k(S^n)$ for all $k \geq 2$.

Problem 5. (May § 9.6 Problem 2) Let $n \geq 3$.

a. Compute the group $\pi_n(\mathbb{R}P^n, \mathbb{R}P^{n-1})$.

b. Deduce that the quotient map of pairs

$$(\mathbb{R}P^n, \mathbb{R}P^{n-1}) \to (\mathbb{R}P^n / \mathbb{R}P^{n-1}, *)$$

does not induce an isomorphism on homotopy groups.

Remark. Feel free to use the facts $\pi_n(S^n) \simeq \mathbb{Z}$ and $\pi_k(S^n) = 0$ for $k < n$ (with $n, k \geq 1$).