Math 527 - Homotopy Theory
Spring 2013
Homework 6, Lecture 2/20

Problem 2. Let (E, e_0) and (B, b_0) be pointed spaces and $p\colon E \to B$ a pointed map. Denote by $F := p^{-1}(b_0)$ the strict fiber of p, and $F(p)$ the homotopy fiber of p, defined as

$$F(p) = \{(e, \gamma) \in E \times B^I \mid \gamma(0) = p(e), \gamma(1) = b_0\}.$$

There is a canonical "inclusion of the strict fiber into the homotopy fiber" $\varphi\colon F \to F(p)$ defined by

$$\varphi(e) = (e, c_{b_0})$$

where $c_{b_0}\colon I \to B$ is the constant path at b_0.

Show that if $p\colon E \to B$ is a fibration, then the map $\varphi\colon F \to F(p)$ is a homotopy equivalence.