

Math 527 - Homotopy Theory
Spring 2013
Homework 7, Lecture 2/25

Problem 1. (May § 10.7 Problem 2)

- a.** Let $f: X \xrightarrow{\sim} Y$ be a weak homotopy equivalence. Assuming X is a CW-complex and Y has the *homotopy type* of a CW-complex, show that f is a homotopy equivalence.
- b.** Show that the space $A := \{0\} \cup \{\frac{1}{n} \mid n \in \mathbb{N}\} \subset \mathbb{R}$ does *not* have the homotopy type of a CW-complex.

Problem 2. Consider the “equatorial” embeddings

$$S^0 \subset S^1 \subset S^2 \subset \dots$$

of spheres, and define the infinite-dimensional sphere $S^\infty := \operatorname{colim}_n S^n$. Show that S^∞ is contractible.