

**Math 527 - Homotopy Theory**  
**Spring 2013**  
**Homework 9, Lecture 3/11**

**Problem 1.** Consider the Hopf map  $\eta: S^3 \rightarrow S^2$ .

**a.** Describe the cofiber  $C(\eta)$ . It is a familiar space.

**b.** Consider the canonical comparison map  $\varphi: F(\eta) \rightarrow \Omega C(\eta)$  from the homotopy fiber to the loop space of the cofiber. Find the lowest dimension  $k$  such that  $\pi_k F(\eta)$  is not isomorphic to  $\pi_k \Omega C(\eta)$  (and thus  $\varphi$  cannot possibly induce an isomorphism on  $\pi_k$ ).

*Remark.* It turns out that  $\varphi$  induces an isomorphism on homotopy groups  $\pi_i$  for  $i < k$ , but you are not asked to show this.