Math 527 - Homotopy Theory
Spring 2013
Homework 9, Lecture 3/15

Problem 3. Let $f\colon X \to Y$ be an n-connected map between spaces, and assume X is m-connected.

a. Using Blakers-Massey, show that the canonical comparison map

$$\varphi\colon F(f) \to \Omega C(f)$$

from the homotopy fiber to the loop space of the cofiber of f is $(m+n)$-connected.

b. Looking back (c.f. Problem 1) at the example of the Hopf map $\eta\colon S^3 \to S^2$, conclude that:

- The connectivity estimate $m+n$ in part (a) cannot be improved in general;
- The map $\varphi_\eta\colon F(\eta) \to \Omega C(\eta)$ does in fact induce isomorphisms on homotopy groups below the least dimension k satisfying $\pi_k F(\eta) \not\cong \pi_k \Omega C(\eta)$.