Math 535 - General Topology
Fall 2012
Homework 13, Lecture 11/28

Note: In this problem set, all function spaces are endowed with the compact-open topology.

Problem 3.

a. Let X and Y be topological spaces, where Y is Hausdorff. Show that $C(X, Y)$ is Hausdorff.

b. Assume there exists a topological space X such that $C(X, Y)$ is Hausdorff. Show that Y is Hausdorff.

Problem 4.

a. Let X, Y, and Z be topological spaces, where X is locally compact Hausdorff. Let $g\colon Y \to Z$ be a continuous map. Show that the induced map "postcomposition by g"

$$\begin{aligned} g_* \colon C(X, Y) &\to C(X, Z) \\ f &\mapsto g_*(f) = g \circ f \end{aligned}$$

is continuous.

b. Let W, X, and Y be topological spaces, where X is locally compact Hausdorff. Let $d\colon W \to X$ be a continuous map. Show that the induced map "precomposition by d"

$$\begin{aligned} d^* \colon C(X, Y) &\to C(W, Y) \\ f &\mapsto d^*(f) = f \circ d \end{aligned}$$

is continuous.

Edit 11/30/2012: The conclusions of 4a and 4b are true in general, without any assumption on the spaces involved.