Math 535 - General Topology
Fall 2012
Homework 13, Lecture 11/30

Problem 5. Let X and Y be topological spaces, where X is *Hausdorff*. Let $\mathcal{S}$ be a subbasis for the topology of Y. Show that the collection

$$\{V(K,S) \mid K \subseteq X \text{ compact}, S \in \mathcal{S}\}$$

is a subbasis for the compact-open topology on $C(X,Y)$.

The notation above is $V(K,S) = \{f \in C(X,Y) \mid f(K) \subseteq S\}$.

Problem 6. Consider the real line $\mathbb{R}$ and the rationals $\mathbb{Q}$ with their standard (metric) topology. Consider the evaluation map

$$e \colon \mathbb{Q} \times C(\mathbb{Q},\mathbb{R}) \to \mathbb{R}.$$

Let $f \colon \mathbb{Q} \to \mathbb{R}$ be a constant function (say, $f \equiv 0$), and let $q \in \mathbb{Q}$. Show that the evaluation map e is *not* continuous at $(q,f) \in \mathbb{Q} \times C(\mathbb{Q},\mathbb{R})$.

Hint: You may want to use the fact that all compact subsets of $\mathbb{Q}$ have empty interior (c.f. Homework 7 Problem 5), and the fact that $\mathbb{Q}$ is completely regular (since it is normal).