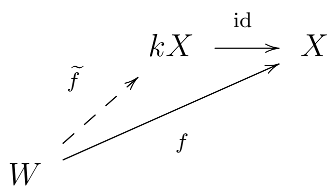
Math 535 - General Topology
Fall 2012
Homework 14, Lecture 12/5

Problem 3. Let X be a topological space and kX its k-ification.

a. Show that the identity function $\mathrm{id}\colon kX \to X$ is a homeomorphism if and only if X is compactly generated.

b. Show that kX is always compactly generated.

c. Let W be a compactly generated space and $f\colon W \to X$ a continuous map. Show that there exists a unique continuous map $\widetilde{f}\colon W \to kX$ satisfying $f = \mathrm{id} \circ \widetilde{f}$, i.e. making the diagram



commute.

Problem 4. Show that any compactly generated space X is a quotient of a coproduct of compact spaces. In other words, there exists a collection $\{K_i\}_{i\in I}$ of compact spaces, indexed by some *set* I, and a quotient map $q\colon \coprod_{i\in I} K_i \twoheadrightarrow X$.