Math 535 - General Topology
Fall 2012
Homework 2, Lecture 9/7

Problem 4. (Munkres Exercise 2.19.7) Consider the set of sequences of real numbers

$$\mathbb{R}^{\mathbb{N}} = \{(x_1, x_2, \ldots) \mid x_n \in \mathbb{R} \text{ for all } n \in \mathbb{N}\} \cong \prod_{n\in\mathbb{N}} \mathbb{R}$$

and consider the subset of sequences that are "eventually zero"

$$\mathbb{R}^{\infty} := \{x \in \mathbb{R}^{\mathbb{N}} \mid x_n \neq 0 \text{ for at most finitely many } n\}.$$

a. In the box topology on $\mathbb{R}^{\mathbb{N}}$, is $\mathbb{R}^{\infty}$ a closed subset?

b. In the product topology on $\mathbb{R}^{\mathbb{N}}$, is $\mathbb{R}^{\infty}$ a closed subset?

Problem 5. Let X be a topological space, S a set, and $f\colon X \to S$ a function. Consider the collection of subsets of S

$$\mathcal{T} := \{U \subseteq S \mid f^{-1}(U) \text{ is open in } X\}.$$

a. Show that $\mathcal{T}$ is a topology on S.

b. Show that $\mathcal{T}$ is the largest topology on S making f continuous.

c. Let Y be a topological space. Show that a map $g\colon S \to Y$ is continuous if and only if the composite $g \circ f\colon X \to Y$ is continuous.

d. Show that $\mathcal{T}$ is the smallest topology on S with the property that a map $g\colon S \to Y$ is continuous whenever $g \circ f$ is continuous.

Problem 6. Consider the subset $X = \{0\} \cup \{\frac{1}{n} \mid n \in \mathbb{N}\} \subset \mathbb{R}$ viewed as a subspace of the real line $\mathbb{R}$. As a *set*, X is the disjoint union of the singletons $\{0\}$ and $\{\frac{1}{n}\}$ for all $n \in \mathbb{N}$. However, show that X does *not* have the coproduct topology on $\{0\} \amalg \coprod_{n\in\mathbb{N}} \{\frac{1}{n}\}$.