Math 535 - General Topology
Fall 2012
Homework 3, Lecture 9/10

Problem 1. Let $S^1 \subset \mathbb{R}^2$ be unit circle in the plane, with the subspace topology. Consider the "winding" map

$$f \colon \mathbb{R} \to S^1$$
$$t \mapsto (\cos t, \sin t).$$

Show that f induces a homeomorphism $\mathbb{R}/\!\sim\, \cong S^1$, where the equivalence relation on $\mathbb{R}$ is $t \sim t'$ if and only if $t - t' = 2k\pi$ for some integer $k \in \mathbb{Z}$.

Problem 2. Let $f \colon X \twoheadrightarrow Y$ be a surjective continuous map.

a. If f is an open map, show that f is a quotient map.

b. If f is a closed map, show that f is a quotient map.

Problem 3. Find an example of a *metric* space X and a *quotient* map $q \colon X \twoheadrightarrow Y$ which is neither an open map nor a closed map.

Note: For the purposes of the homework, Y is not required to be a metric space, although such examples can be found.

Problem 4. (Bredon Exercise I.13.6) Consider the quotient space $\mathbb{R}/\mathbb{Q}$, where the equivalence relation on $\mathbb{R}$ is $x \sim x'$ if and only if $x - x' \in \mathbb{Q}$. Show that the topology on $\mathbb{R}/\mathbb{Q}$ is anti-discrete, i.e. only the empty set $\emptyset$ and all of $\mathbb{R}/\mathbb{Q}$ are open.