Math 535 - General Topology
Fall 2012
Homework 4, Lecture 9/19

Problem 4. (Brown Exercise 3.5.3) Prove that a discrete space is compact if and only if it is finite.

Problem 5. (Munkres Exercise 3.26.2) Let X be a set endowed with the cofinite topology. Show that every subspace $A \subseteq X$ is compact.

Problem 6. (Munkres Exercise 3.26.5) (Willard Exercise 6.17B.5) Let X be a *Hausdorff* topological space.

a. Let $A \subset X$ be a *compact* subspace and $x_0 \in X \setminus A$ a point outside A. Show that A and x_0 can be separated by neighborhoods, i.e. there exist open subsets $U, V \subset X$ satisfying $A \subseteq U$, $x_0 \in V$, and $U \cap V = \emptyset$.

b. Let $A, B \subset X$ be disjoint *compact* subspaces. Show that A and B can be separated by neighborhoods, i.e. there exist open subsets $U, V \subset X$ satisfying $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$.