Math 535 - General Topology
Fall 2012
Homework 4, Lecture 9/21

Problem 7. Let D^n denote the unit disc in $\mathbb{R}^n$ (with the usual Euclidean norm)

$$D^n := \{x \in \mathbb{R}^n \mid \|x\| \leq 1\}$$

and let S^n denote the unit sphere in $\mathbb{R}^{n+1}$

$$S^n := \{x \in \mathbb{R}^{n+1} \mid \|x\| = 1\}.$$

Show that there is a homeomorphism

$$(D^n \amalg D^n)/\sim \; \cong S^n$$

where the two discs are glued along their edges, i.e. the equivalence relation $\sim$ is generated by $x^{(1)} \sim x^{(2)}$ for all $x \in D^n$ with $\|x\| = 1$. Here the superscript denotes that $x^{(1)} \in D^n \amalg D^n$ lives in the first summand while $x^{(2)}$ lives in the second summand.

Problem 8. (Munkres Exercise 3.26.4)

a. Let (X, d) be a metric space, and $K \subseteq X$ a compact subspace. Show that K is closed (in X) and bounded.

Now we show that the converse does not hold.

b. Find a metric space (X, d) and a subset $C \subseteq X$ which is closed and bounded, but such that C is *not* compact.