Math 535 - General Topology
Fall 2012
Homework 7, Lecture 10/10

Problem 3. Show that a topological space X is Tychonoff (a.k.a. $T_{3\frac{1}{2}}$) if and only if X is homeomorphic to a subspace of a cube

$$[0,1]^I \cong \prod_{i \in I} [0,1]$$

where I is an arbitrary indexing set.

Problem 4. For parts (a) and (b), let X and Y be topological spaces, where Y is *Hausdorff*.

a. Let $f, g \colon X \to Y$ be two continuous maps. Show that the subset

$$E := \{x \in X \mid f(x) = g(x)\}$$

where the two maps agree is closed in X.

b. Let $f, g \colon X \to Y$ be two continuous maps and assume $D \subseteq X$ is a dense subset on which the two maps agree, i.e. $f|_D = g|_D$. Show that the two maps agree everywhere, i.e. $f = g$.

c. Find an example of a *metric* space X along with a dense subset $D \subset X$ and a continuous map $f \colon D \to [0,1]$ that does *not* admit a continuous extension to all of X.

d. Let X be a *separable* topological space. Show that the set $C(X, \mathbb{R})$ of all continuous real-valued functions on X satisfies the cardinality bound

$$|C(X,\mathbb{R})| \leq |\mathbb{R}|^{\aleph_0}$$

where $\aleph_0 = |\mathbb{N}|$ is the countably infinite cardinal.

Recall: A topological space is **separable** if it has a countable dense subset.