Math 535 - General Topology
Fall 2012
Homework 7, Lecture 10/12

Problem 5. (Munkres Exercise 29.1) Show that the space $\mathbb{Q}$ of rational numbers, with its standard topology, is *not* locally compact.

Problem 6. Let X be a set. The **particular point topology** on X with "particular point" $p \in X$ is defined as

$$\mathcal{T} = \{S \subseteq X \mid p \in S \text{ or } S = \emptyset\}.$$

One readily checks that $\mathcal{T}$ is indeed a topology.

a. Show that X (endowed with the particular point topology) is locally compact.

b. Show that X is compact if and only if X is finite.

c. Show that X is Lindelöf if and only if X is countable.

d. Assuming X is uncountable, find a *compact* subspace $K \subseteq X$ whose closure $\overline{K}$ is not compact, in fact not even Lindelöf.