Math 535 - General Topology
Fall 2012
Homework 9, Lecture 10/22

Problem 1.

a. Let X be a topological space with finitely many connected components. Show that each connected component is open in X.

b. Let X be a topological space and $\{U_i\}_{i\in I}$ a collection of open subsets of X such that X is the disjoint union $X = \bigsqcup_{i\in I} U_i$. Show that X is the coproduct $X = \coprod_{i\in I} U_i$.

In particular, this conclusion applies to the situation in part (a).

c. Find an example of *metrizable* space X with a connected component $C \subset X$ which is *not open* in X.

Problem 2. Show that the n-dimensional sphere S^n is path-connected (for $n \geq 1$).