Math 535 - General Topology
Fall 2012
Homework 9, Lecture 10/26

Problem 5. Let $p \in \mathbb{R}^n$. Show that $\mathbb{R}^n \setminus \{p\}$ is homotopy equivalent to the $(n-1)$-dimensional sphere S^{n-1}.

Problem 6. Let X be a topological space and denote by $\pi_0(X)$ the set of path components of X.

a. Show that any continuous map $f \colon X \to Y$ induces a well-defined function

$$\pi_0(f) \colon \pi_0(X) \to \pi_0(Y).$$

b. Show that the induced function $\pi_0(f)$ only depends on the homotopy class of f. In other words, if $f \simeq f'$ are homotopic maps, then $\pi_0(f) = \pi_0(f')$.

c. Show that a homotopy equivalence $f \colon X \xrightarrow{\simeq} Y$ induces a bijection $\pi_0(f) \colon \pi_0(X) \xrightarrow{\simeq} \pi_0(Y)$.

Remark. This proves in particular that path-connectedness is a homotopy invariant. Given homotopy equivalent spaces X and Y, then X is path-connected if and only if Y is path-connected.