

Math 9052B/4152B - Algebraic Topology
Winter 2015
Topics for presentations

Martin Frankland

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Here are a few suggested topics for the oral presentations. The references will be to Hatcher [4] unless otherwise noted.

1. Cellular approximation theorem. §4.1
2. Van Kampen theorem with groupoids. [3, §6.7, 9.1] [6, Chapter 17]
3. Higher homotopy groups. Long exact sequence of a pair. Fiber sequences. §4.1
4. Eilenberg-MacLane spaces and Postnikov towers. §4.2, 4.3
5. Hurewicz theorem. §2.A for the case $n = 1$; §4.2 for the general statement.
6. Euler characteristic. Poincaré-Hopf index theorem. §2.2, [2, §IV.13, VI.12]
7. Lefschetz trace formula and fixed-point theorem. §2.C, [2, §IV.23]
8. Homology of manifolds. Orientation. Fundamental class. §3.3, [2, §VI.7]
9. Universal coefficient theorem for homology. §3.A, [2, §V.7]
10. Algebraic Künneth theorem. (Note that the universal coefficient theorem is a special case.) §3.2, 3.B, [2, §VI.1]
11. Eilenberg-Zilber theorem and topological Künneth theorem. §3.2, 3.B, [2, §IV.16, VI.1]
12. Homology with twisted coefficients. Homology of covering spaces. §3.H
13. Universal coefficient theorem for cohomology. §3.1, [2, §V.7]
14. The cup product, with examples and applications. §3.2
15. Mod 2 cohomology of $\mathbb{R}P^\infty$ and integral cohomology of $\mathbb{C}P^\infty$. §3.2

16. Classification of surfaces. [7, §1.5 - 1.7]
17. Poincare duality. §3.3
18. De Rham cohomology. [2, §V.1 - V.5, V.9] [1, §1.1 - 1.5]
19. Fiber bundles and fibrations. Hopf fibration. §4.2
20. The Leray-Hirsch theorem and Thom isomorphism. §4.D, [8, §10]
21. Vector bundles and characteristic classes. [5, Chapters 1 and 3] [2, §VI.17] [8, §2 - 5]

References

- [1] R. Bott and L. W. Tu, *Differential forms in algebraic topology*, Graduate Texts in Mathematics, vol. 82, Springer-Verlag, New York-Berlin, 1982.
- [2] G. E. Bredon, *Topology and geometry*, Graduate Texts in Mathematics, vol. 139, Springer-Verlag, New York, 1997. Corrected third printing of the 1993 original.
- [3] R. Brown, *Topology and groupoids*, BookSurge, LLC, Charleston, SC, 2006. Third edition.
- [4] A. Hatcher, *Algebraic topology*, Cambridge University Press, Cambridge, 2002.
- [5] ———, *Vector bundles and K-theory*, 2009, Preprint.
- [6] P. J. Higgins, *Categories and groupoids*, Repr. Theory Appl. Categ. **7** (2005), 1–178. Reprint of the 1971 original.
- [7] W. S. Massey, *Algebraic topology: an introduction*, Springer-Verlag, New York-Heidelberg, 1977. Reprint of the 1967 edition; Graduate Texts in Mathematics, Vol. 56.
- [8] J. W. Milnor and J. D. Stasheff, *Characteristic classes*, Princeton University Press, Princeton, N. J.; University of Tokyo Press, Tokyo, 1974. Annals of Mathematics Studies, No. 76.