The Drude Model

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Paul Drude, German physicist, 1863-1906
The Drude Model

Overview

Model

Dielectric medium

Permittivity of metals

Electrical conductors

Faraday effect

Hall effect
• The Drude model links optical and electric properties of a material with the behavior of its electrons or holes
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- Dielectric permittivity
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Consider a typical electron denoted by $x(t)$ the deviation from its equilibrium position.

External electric field strength $E(t)$

Electron mass $m$, charge $q$, friction coefficient $m\Gamma$, spring constant $m\Omega^2$

Fourier transform this

Solution is $	ilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}(\omega)}{\Omega^2 - \omega^2 - i\omega\Gamma}$
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  $$\tilde{x}(\omega) = \frac{q}{m \Omega^2 - \omega^2 - i\omega\Gamma} \tilde{E}(\omega)$$
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Polarization

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Polarization

• dipole moment of typical electron is

\[ \tilde{p} = q \tilde{x} \]

• recall

\[ \tilde{x}(\omega) = \frac{q m \tilde{E}(\omega)}{\Omega^2 - \omega^2 - i \omega \Gamma} \]

• there are \( N \) typical electrons per unit volume

• polarization is

\[ \tilde{P} = Nq \tilde{x} = \epsilon_0 \chi \tilde{E} \]

• susceptibility is

\[ \chi(\omega) = \frac{Nq^2 \epsilon_0 m}{\Omega^2 - \omega^2 - i \omega \Gamma} \]

• in particular

\[ \chi(0) = \frac{Nq^2 \epsilon_0 m}{\Omega^2} > 0 \]

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Discussion

1

• decompose susceptibility \( \chi(\omega) = \chi'(\omega) + i \chi''(\omega) \) into refractive part \( \chi' \) and absorptive part \( \chi'' \)

• Introduce \( R(\omega) = \frac{\chi(\omega)}{\chi(0)} \), \( s = \frac{\omega}{\Omega} \) and \( \gamma = \frac{\Gamma}{\Omega} \) as normalized quantities.

• refraction \( R'(s) = 1 - s^2 \frac{(1 - s^2)^2 + \gamma^2 s^2}{(1 - s^2)^2 + \gamma^2 s^2} \)

• absorption \( D''(s) = s \frac{(1 - s^2)^2 + \gamma^2 s^2}{(1 - s^2)^2 + \gamma^2 s^2} + \gamma s \)

• limiting cases: \( s = 0, s = 1, s \to \infty \), small \( \gamma \)
• decompose susceptibility $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$ into refractive part $\chi'$ and absorptive part $\chi''$
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• limiting cases: \( s = 0, s = 1, s \to \infty, \text{small } \gamma \)
Refractive part (blue) and absorptive part (red) of the susceptibility function $\chi(\omega)$ scaled by the static value $\chi(0)$. The abscissa is $\omega/\Omega$. $\Gamma/\Omega = 0.1$
Discussion II

- For small frequencies (as compared with $\omega$), the susceptibility is practically real.
- This is the realm of classical optics
- $\partial \chi / \partial \omega$ is positive – normal dispersion
- In the vicinity of $\omega = \Omega$ absorption is large. Negative dispersion $\partial \chi / \partial \omega$ is accompanied by strong absorption.
- For very large frequencies again absorption is negligible, and the susceptibility is negative with normal dispersion. This applies to X rays.
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• $\chi(\infty) = 0$ is required by first principles . . .
Kramers-Kronig relation I

\[ \chi(\omega) \text{ must be the Fourier transform of a causal response function } G(\tau) = G(\tau) \]

\[ P(t) = \epsilon_0 \int d\tau G(\tau) E(t - \tau) \]

• poles at \( \omega_1, 2 = -\frac{i}{2} \pm \bar{\omega} \)

\[ \bar{\omega} = \sqrt{\Omega^2 - \Gamma^2}/4 \]

Indeed, \( G(\tau) = 0 \) for \( \tau < 0 \)

\[ G(\tau) = Nq^2 \epsilon_0 m \sin \bar{\omega} \tau \bar{\omega} e^{-\Gamma \tau/2} \]
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• for \( \tau > 0 \)

\[
G(\tau) = \frac{Nq^2}{\epsilon_0 m} \frac{\sin \bar{\omega}\tau}{\bar{\omega}} e^{-\Gamma\tau/2}
\]
Kramers-Kronig relation II

• causal response function:
  \[ G(\tau) = \theta(\tau) G(\tau) \]

• apply the convolution theorem
  \[ \chi(\omega) = \int \frac{d\omega}{2\pi} \chi(u) \tilde{\theta}(\omega - u) \]

• Fourier transform of Heaviside function is
  \[ \tilde{\theta}(\omega) = \lim_{\eta \to 0} \frac{1}{\eta - i\omega} \]

• dispersion, or Kramers-Kronig relations
  \[ \chi'(\omega) = 2\text{Pr} \int \frac{du}{\pi} u \chi''(u) \left( u^2 - \omega^2 \right) \]
  \[ \chi''(\omega) = 2\text{Pr} \int \frac{du}{\pi} \omega \chi'(u) \left( \omega^2 - u^2 \right) \]
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  \chi''(\omega) = 2\text{Pr} \int \frac{du}{\pi} \frac{\omega\chi'(u)}{\omega^2 - u^2}
  \]
Dispersion of white light
Overview of the Drude model: a model for dielectric media, including permittivity of metals, electrical conductors, Faraday effect, and Hall effect. Free quasi-electrons are considered as free quasi-particles, with their behavior governed by the equation:

\[
m \left(\ddot{x} + \Gamma \dot{x} + \Omega^2 x\right) = qE
\]

The spring constant \(m\) vanishes, and the effective mass \(m\) is determined. The permittivity \(\epsilon(\omega)\) is given by:

\[
\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} + i\frac{\omega \Gamma}{\omega_p^2}
\]

Where \(\omega_p\) is the plasma frequency, \(\epsilon_0\) is the permittivity of free space, and \(\omega\) is the angular frequency.

Corrections for \(\omega \gg \omega_p\) are applied.

References and further reading are included.
Free quasi-electrons

- consider a typical conduction band electron
Free quasi-electrons

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- it behaves as a free quasi-particle
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Free quasi-electrons

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  \epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}
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  \omega_p^2 = \frac{Nq^2}{\epsilon_0 m}
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  \[
  \epsilon(\omega) = \epsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}
  \]
Example: gold

Drude model parameters for gold as determined by Johnson and Christy in 1972:

- $\varepsilon_\infty = 9.5$
- $\hbar \omega_p = 8.95 \text{ eV}$
- $\hbar \Gamma = 0.069 \text{ eV}$

With these parameters, the Drude model fits optical measurements well for $\hbar \omega < 2.25 \text{ eV}$ (green).

The refractive part of the permittivity can be large and negative while the absorptive part is small. This allows surface plasmon polaritons (SPP).
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- The refractive part of the permittivity can be large and negative while the absorptive part is small.
- This allows surface plasmon polaritons (SPP)
Refractive (blue) and absorptive part (red) of the permittivity function for gold. The abscissa is $\hbar \omega$ in eV.
Electrical conductivity

Consider a typical charged particle:

\[ m \ddot{x} + \Gamma \dot{x} + \Omega^2 x = qE \]

Electric current density:

\[ J = Nq \dot{x} \]

Fourier transformed:

\[ \tilde{J} = Nq (\omega^2 - i\omega) \tilde{x} \]

\[ \tilde{x}(\omega) = qm \tilde{E}(\omega) \Omega^2 - \omega^2 - i\omega \Gamma \]

Ohm's law:

\[ \tilde{J} = \sigma(\omega) \tilde{E} \]

Conductivity:

\[ \sigma(\omega) = \frac{Nq^2}{m} - \frac{i\omega \Omega^2}{\omega^2 - i\omega \Gamma} \]
• consider a typical charged particle
• consider a typical charged particle
• recall $m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = qE$
• consider a typical charged particle
• recall \( m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = qE \)
• electric current density \( J = Nq\dot{x} \)
• consider a typical charged particle

• recall  \( m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = qE \)

• electric current density  \( \mathbf{J} = Nq \dot{x} \)

• Fourier transformed:  \( \tilde{\mathbf{J}} = Nq(-i\omega)\tilde{x} \)
• consider a typical charged particle
• recall \( m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = qE \)
• electric current density \( J = Nq\dot{x} \)
• Fourier transformed: \( \tilde{J} = Nq(-i\omega)\tilde{x} \)
• recall

\[
\tilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}(\omega)}{\Omega^2 - \omega^2 - i\omega\Gamma}
\]
• consider a typical charged particle
• recall $m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = qE$
• electric current density $J = Nq\dot{x}$
• Fourier transformed: $\tilde{J} = Nq(-i\omega)\tilde{x}$
• recall

\[
\tilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}(\omega)}{\Omega^2 - \omega^2 - i\omega\Gamma}
\]

• **Ohm’s law**

\[
\tilde{J}(\omega) = \sigma(\omega)\tilde{E}(\omega)
\]
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Hall effect

Electrical conductivity

- consider a typical charged particle
- recall $m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = qE$
- electric current density $J = Nq\dot{x}$
- Fourier transformed: $\tilde{J} = Nq(−i\omega)\tilde{x}$
- recall
  $$\tilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}(\omega)}{\Omega^2 − \omega^2 − i\omega\Gamma}$$
- Ohm’s law
  $$\tilde{J}(\omega) = \sigma(\omega) \tilde{E}(\omega)$$
- conductivity is
  $$\sigma(\omega) = \frac{Nq^2}{m} \frac{−i\omega}{\Omega^2 − \omega^2 − i\omega\Gamma}$$
Electrical conductors

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• A material with $\sigma(0) = 0$ is an electrical insulator. It cannot transport direct currents (DC).
Electrical conductors

- A material with $\sigma(0) = 0$ is an **electrical insulator**. It cannot transport direct currents (DC).
- A material with $\sigma(0) > 0$ is an electrical **conductor**.
Electrical conductors

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• A material with $\sigma(0) = 0$ is an electrical insulator. It cannot transport direct currents (DC).
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$$\sigma(\omega) = \frac{Nq^2}{m} \frac{1}{\Gamma - i\omega}$$
A material with \( \sigma(0) = 0 \) is an electrical insulator. It cannot transport direct currents (DC).

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Charged particles must be free, \( \Omega = 0 \).

which means

\[
\sigma(\omega) = \frac{Nq^2}{m} \frac{1}{\Gamma - i\omega}
\]

or

\[
\frac{\sigma(\omega)}{\sigma(0)} = \frac{1}{1 - i\omega/\Gamma}
\]
• A material with $\sigma(0) = 0$ is an electrical insulator. It cannot transport direct currents (DC).
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• which means
  \[ \sigma(\omega) = \frac{Nq^2}{m} \frac{1}{\Gamma - i\omega} \]
• or
  \[ \frac{\sigma(\omega)}{\sigma(0)} = \frac{1}{1 - i\omega/\Gamma} \]
• Note that the DC conductivity is always positive.
Georg Simon Ohm, German physicist, 1789-1854
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External static magnetic field
• apply a quasi-static external induction $\mathbf{B}$
External static magnetic field

- apply a quasi-static external induction $\mathcal{B}$
- the typical electron obeys

$$m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = q(E + \dot{x} \times \mathcal{B})$$
External static magnetic field

- apply a quasi-static external induction $B$
- the typical electron obeys
  \[ m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = q(E + \dot{x} \times B) \]
- Fourier transform this
  \[ m(-\omega^2 - i\omega \Gamma + \Omega^2)\tilde{x} = q(\tilde{E} - i\omega \tilde{x} \times B) \]
apply a quasi-static external induction $\mathcal{B}$

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$$m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = q(E + \dot{x} \times \mathcal{B})$$

Fourier transform this

$$m(-\omega^2 - i\omega\Gamma + \Omega^2)\tilde{x} = q(\tilde{E} - i\omega\tilde{x} \times \mathcal{B})$$

assume $\mathcal{B} = B\hat{e}_z$
External static magnetic field

- apply a quasi-static external induction $\mathcal{B}$
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  \[ m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = q(E + \dot{x} \times \mathcal{B}) \]
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  \[ m(-\omega^2 - i\omega \Gamma + \Omega^2)\tilde{x} = q(\tilde{E} - i\omega \tilde{x} \times \mathcal{B}) \]
- assume $\mathcal{B} = B\hat{e}_z$
- assume circularly polarized light
  \[ \tilde{E} = \tilde{E}_\pm \hat{e}_\pm \text{ where } \hat{e}_\pm = (\hat{e}_x + i\hat{e}_y)/\sqrt{2} \]
• apply a quasi-static external induction $\mathcal{B}$
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  \[ \tilde{E} = \tilde{E}_\pm \hat{e}_\pm \text{ where } \hat{e}_\pm = (\hat{e}_x + i\hat{e}_y)/\sqrt{2} \]
• try $\tilde{x} = \tilde{x}_\pm \hat{e}_\pm$
apply a quasi-static external induction $\mathcal{B}$
the typical electron obeys
\[
m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = q(E + \dot{x} \times \mathcal{B})
\]
Fourier transform this
\[
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assume $\mathcal{B} = B\hat{e}_z$
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try \(\tilde{x} = \tilde{x}_\pm \hat{e}_\pm\)
note \(\hat{e}_\pm \times \hat{e}_z = \mp i\hat{e}_\pm\)
apply a quasi-static external induction $\mathcal{B}$

the typical electron obeys

$$m(\ddot{x} + \Gamma \dot{x} + \Omega^2 x) = q(E + \dot{x} \times \mathcal{B})$$

Fourier transform this

$$m(-\omega^2 - i \omega \Gamma + \Omega^2)\tilde{x} = q(\tilde{E} - i \omega \tilde{x} \times \mathcal{B})$$

assume $\mathcal{B} = B\hat{e}_z$

assume circularly polarized light

$$\tilde{E} = \tilde{E}_\pm \hat{e}_\pm \text{ where } \hat{e}_\pm = (\hat{e}_x + i \hat{e}_y)/\sqrt{2}$$

try $\tilde{x} = \tilde{x}_\pm \hat{e}_\pm$

note $\hat{e}_\pm \times \hat{e}_z = \mp i \hat{e}_\pm$

therefore

$$m(-\omega^2 - i \omega \Gamma + \Omega^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega B \tilde{x}_\pm)$$
Faraday effect
\begin{itemize}
\item \( m(-\omega^2 - i\omega\Gamma + \Omega^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega B\tilde{x}_\pm) \)
\end{itemize}
$m(-\omega^2 - i\omega\Gamma + \Omega^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega B \tilde{x}_\pm)$

therefore

$$\tilde{x}_\pm = \frac{q\tilde{E}_\pm}{m(\Omega^2 - i\omega\Gamma - \omega^2) \pm q\omega B}$$
• \( m(-\omega^2 - i\omega\Gamma + \Omega^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega B\tilde{x}_\pm) \)
• therefore
\[
\tilde{x}_\pm = \frac{q\tilde{E}_\pm}{m(\Omega^2 - i\omega\Gamma - \omega^2) \pm q\omega B}
\]
• recall \( \tilde{P} = Nq\tilde{x} = \epsilon_0\chi\tilde{E} \)
Faraday effect

\begin{itemize}
  \item \(m(-\omega^2 - i\omega \Gamma + \Omega^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega B \tilde{x}_\pm)\)
  
  \item therefore
  \[
  \tilde{x}_\pm = \frac{q\tilde{E}_\pm}{m(\Omega^2 - i\omega \Gamma - \omega^2) \pm q\omega B}
  \]
  
  \item recall \(\tilde{P} = Nq\tilde{x} = \epsilon_0 \chi \tilde{E}\)
  
  \item effect of quasi-static induction \(B\) is
  \[
  \chi_{\pm}(\omega) = \frac{Nq^2}{\epsilon_0 m} \frac{1}{\Omega^2 - i\omega \Gamma - \omega^2 \pm (q/m)\omega B}
  \]
\end{itemize}
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- \[ m(-\omega^2 - i\omega \Gamma + \Omega^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega \mathcal{B}\tilde{x}_\pm) \]

- therefore

\[ \tilde{x}_\pm = \frac{q\tilde{E}_\pm}{m(\Omega^2 - i\omega \Gamma - \omega^2) \pm q\omega \mathcal{B}} \]

- recall \( \tilde{P} = Nq\tilde{x} = \epsilon_0 \chi \tilde{E} \)

- effect of quasi-static induction \( \mathcal{B} \) is

\[ \chi_\pm(\omega) = \frac{Nq^2}{\epsilon_0 m \Omega^2 - i\omega \Gamma - \omega^2 \pm (q/m)\omega \mathcal{B}} \]

- left and right handed polarized light sees different susceptibility
\[ m(-\omega^2 - i\omega \Gamma + \Omega^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega \mathcal{B}\tilde{x}_\pm) \]

therefore

\[ \tilde{x}_\pm = \frac{q\tilde{E}_\pm}{m(\Omega^2 - i\omega \Gamma - \omega^2) \pm q\omega \mathcal{B}} \]

recall \( \tilde{P} = Nq\tilde{x} = \epsilon_0\chi \tilde{E} \)

effect of quasi-static induction \( \mathcal{B} \) is

\[ \chi_\pm(\omega) = \frac{Nq^2}{\epsilon_0 m \frac{1}{\Omega^2 - i\omega \Gamma - \omega^2 \pm (q/m)\omega \mathcal{B}}} \]

left and right handed polarized light sees different susceptibility

Faraday effect
Michael Faraday, English physicist, 1791-1867
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Remarks

- $\mathbf{B}$ is always small (in natural units)
- $\varepsilon_{ij}(\omega; B) = \varepsilon_{ij}(\omega; 0) + iK(\omega)\varepsilon_{ijk}B_k$
- linear magneto-optic effect
- Faraday constant is $K(\omega) = Nq^3\varepsilon_0m^2\omega(\Omega^2 - i\omega\Gamma - \omega^2)^2$
- $K(\omega)$ is real in transparency window
- i.e. if $\omega$ is far away from $\Omega$
- Faraday effect distinguishes between forward and backward propagation

Optical isolator
• $\mathcal{B}$ is always small (in natural units)
• $B$ is always small (in natural units)

• $\varepsilon_{ij}(\omega; B) = \varepsilon_{ij}(\omega; 0) + i K(\omega)\varepsilon_{ijk}B_k$
Remarks

- $B$ is always small (in natural units)
- $\varepsilon_{ij}(\omega; B) = \varepsilon_{ij}(\omega; 0) + i K(\omega) \varepsilon_{ijk} B_k$
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• \( B \) is always small (in natural units)

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• linear magneto-optic effect

• Faraday constant is

\[
K(\omega) = \frac{Nq^3}{\epsilon_0 m^2} \frac{\omega}{(\Omega^2 - i\omega\Gamma - \omega^2)^2}
\]
- **Remarks**

  - $\mathcal{B}$ is always small (in natural units)
  - $\epsilon_{ij}(\omega; \mathcal{B}) = \epsilon_{ij}(\omega; 0) + i K(\omega) \epsilon_{ijk} \mathcal{B}_k$
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  - Faraday constant is
    \[
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  • $K(\omega)$ is real in transparency window
  • i. e. if $\omega$ is far away from $\Omega$
  • Faraday effect distinguishes between forward and backward propagation
  • optical isolator
Conduction in a magnetic field

Peter Hertel

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\[ m \frac{\omega^2 - i \Gamma \omega}{\omega (\omega^2 - i \Gamma \omega)} \tilde{x} = q (\tilde{E} - i \omega \tilde{x} \times \tilde{B}) \]

\[ \text{Ohmic current } \propto \tilde{E} \text{ and Hall current } \propto \tilde{E} \times \tilde{B} \]
Conduction in a magnetic field

- set the spring constant $m\Omega^2 = 0$
Conduction in a magnetic field

- set the spring constant $m\Omega^2 = 0$
- study AC electric field $\tilde{E}$
Conduction in a magnetic field

- set the spring constant $m\Omega^2 = 0$
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Conduction in a magnetic field

- set the spring constant $m\Omega^2 = 0$
- study AC electric field $\tilde{E}$
- and static magnetic induction $B$
- solve
  \[ m(-\omega^2 - i\Gamma\omega)\tilde{x} = q(\tilde{E} - i\omega\tilde{x} \times B) \]
Conduction in a magnetic field

- set the spring constant $m\Omega^2 = 0$
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- solve
  \[ m(-\omega^2 - i\Gamma \omega)\tilde{x} = q(\tilde{E} - i\omega \tilde{x} \times \mathcal{B}) \]
- or
  \[ \tilde{x} = \frac{q}{m - i\omega \Gamma - i\omega} \left\{ \tilde{E} - i\omega \tilde{x} \times \mathcal{B} \right\} \]
Conduction in a magnetic field

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  m(-\omega^2 - i\Gamma\omega)\tilde{x} = q(\tilde{E} - i\omega\tilde{x} \times \mathcal{B})
  \]
- or
  \[
  \tilde{x} = \frac{q}{m - i\omega \Gamma - i\omega} \left\{ \tilde{E} - i\omega\tilde{x} \times \mathcal{B} \right\}
  \]
- by iteration
  \[
  \tilde{x} = \ldots \left\{ \tilde{E} + \frac{q}{m \Gamma - i\omega} \tilde{E} \times \mathcal{B} \right\}
  \]
Conduction in a magnetic field

- set the spring constant \( m\Omega^2 = 0 \)
- study AC electric field \( \tilde{E} \)
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  \]
- or
  \[
  \tilde{x} = \frac{q}{m - i\omega} \left( \frac{1}{\Gamma - i\omega} \{\tilde{E} - i\omega\tilde{x} \times B\} \right)
  \]
- by iteration
  \[
  \tilde{x} = \ldots \left\{ \tilde{E} + \frac{q}{m} \frac{1}{\Gamma - i\omega} \tilde{E} \times B \right\}
  \]
- Ohmic current \( \propto \tilde{E} \) and Hall current \( \propto \tilde{E} \times B \)
Hall effect, schematically
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Hall effect

- Hall current usually forbidden by boundary conditions
- Hall field \( \tilde{E}_H = -\frac{q}{m_1} \Gamma - i\omega \tilde{E} \times B \)
- replace \( \tilde{E} \) by \( \tilde{E} + \tilde{E}_H \)
- \( \tilde{E}_H \times B \) can be neglected
- current \( \tilde{J}(\omega) = \sigma(\omega) \tilde{E}(\omega) \) as usual
- additional Hall field \( \tilde{E}_H(\omega) = R(\omega) \tilde{J}(\omega) \times B \)
- Hall constant \( R = -1/Nq \) does not depend on \( \omega \).
- ... if there is a dominant charge carrier.
- \( R \) has different sign for electrons and holes.
• Hall current usually forbidden by boundary conditions
Hall effect

- Hall current usually forbidden by boundary conditions
- Hall field
  \[
  \tilde{\mathcal{E}}_H = -\frac{q}{m \Gamma - i\omega} \tilde{\mathcal{E}} \times \mathcal{B}
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• Hall current usually forbidden by boundary conditions
• Hall field
\[ \tilde{\mathcal{E}}_H = -\frac{q}{m \Gamma - i\omega} \tilde{\mathcal{E}} \times \mathcal{B} \]
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• Hall current usually forbidden by boundary conditions

• Hall field
\[ \tilde{E}_H = -\frac{q}{m \Gamma - i\omega} \tilde{E} \times B \]

• replace \( \tilde{E} \) by \( \tilde{E} + \tilde{E}_H \)

• \( \tilde{E}_H \times B \) can be neglected

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Hall effect

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  \[ \tilde{E}_H = -\frac{q}{m \Gamma - i\omega} \tilde{E} \times B \]
  - replace \( \tilde{E} \) by \( \tilde{E} + \tilde{E}_H \)
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- ... if there is a dominant charge carrier.
• Hall current usually forbidden by boundary conditions
• Hall field
  \[ \tilde{E}_H = - \frac{q}{m \Gamma - i\omega} \tilde{E} \times B \]
• replace \( \tilde{E} \) by \( \tilde{E} + \tilde{E}_H \)
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• \( \ldots \) if there is a dominant charge carrier.
• \( R \) has different sign for electrons and holes