Corrections to

Cohen–Macaulay rings

Revised Edition

W. Bruns and J. Herzog

If you should find a mistake in the first or revised edition of our book, mathematical or typographical, please let us know by e-mail to

`wbruns@uos.de`

However, we will list only those mistakes that have not been corrected in the revised edition. All data below refer to it.

---

28 July 2009

---

p. 7, l. −2 $R = k[X, Y, Z]$

p. 9, l. −7 $\operatorname{Ext}_R^{n-1}(N, M/x_1M)$

p. 17, l. 9 Add a closing round bracket before the period.

p. 33, l. 11 for all i → for all n

p. 39, 1.5.21 I contains an R-regular sequence of length 2, but no such sequence of length 3,

p. 62, l. 3 (maximal) → (the maximal possible)

p. 65, 2.1.29 One should additionally suppose that R is a graded $\mathbb{Z}$-algebra with $R_0 = \mathbb{Z}$. Then every homogeneous maximal ideal in $R \otimes \mathbb{Z}_\mathfrak{p}$ contains p. Note that a graded ring R is Cohen–Macaulay if and only if its localizations with respect to graded maximal ideals are Cohen–Macaulay (see 2.1.27). Later on 2.1.29 is only applied in situations where R is even a positively graded $\mathbb{Z}$-algebra.

p. 79, 2.3.10 Part (a) should be augmented by the statement that $I : \Delta = \mathfrak{n}$. This fact is used in the proof of 10.3.5 (see below).The proof covers the additional claim, which amounts to $\Delta S/I \cong S/\mathfrak{n}$. It is shown that $\Delta S/I$ is isomorphic to $H_n(\boldsymbol{y}, S/I)$, and $H_n(\boldsymbol{y}, S/I)$ is isomorphic to $S/\mathfrak{n}$ by 2.3.9 (or via the identification $H_n(\boldsymbol{y}, S/I) \cong \operatorname{Tor}_n^R(S/\mathfrak{n}, S/I) \cong H_n(\boldsymbol{a}, S/\mathfrak{n}) \cong S/\mathfrak{n}$.)

p. 91, 3.1.5(b) principal ideal domain

p. 95, 3rd paragraph The proof can be shortened by a direct appeal to 1.2.4.

p. 105, proof of 3.2.13, l. 2 then $\operatorname{Supp}(Rx) \subset \operatorname{Supp} E = \{\mathfrak{m}\}$, see 3.2.5.

p. 108, l. −2 assumtion → assumption

p. 110, l. 7–13 Replace these lines by the following:
and so $\operatorname{Hom}_R(C, C')$ is cyclic by Nakayama's lemma. Let φ be a generator of this module. Then the natural epimorphism $R \to \operatorname{Hom}_R(C, C')$, $1 \mapsto \varphi$, induces the above isomorphism modulo $\boldsymbol{x}$. By 3.3.3, $\operatorname{Hom}_R(C, C')$ is a maximal Cohen–Macaulay module. Thus 3.3.2 implies that $R \to \operatorname{Hom}_R(C, C')$ is an isomorphism.

p. 112, l. 6–12 These lines contain an argument showing that multiplication by $a \neq 0$ cannot be the zero map on $E_R(k)$. The argument can be replaced by a reference to 3.2.12(e)(ii).

p. 118, 3.3.22 Replace the second (e) by (f).

p. 129, 3.4.5(b) One has $H^i_{\mathfrak{m}}(M) = 0$ if $i < \operatorname{depth} M$.

p. 135, 3.5.12(b) for all $i \geq \dim R$

p. 138, 3.6.3(b) $\mathfrak{p} \in R \to \mathfrak{p} \subset R$

p. 139, 3.6.9 The condition in (b) "provided $\mathfrak{m}$ is maximal" is superfluous. In fact, if $R/\mathfrak{m} \cong (R/\mathfrak{m})(-i)$, then there exists a homogeneous unit of degree i, and so every graded R-module M is isomorphic to $M(-i)$ in $\mathscr{M}_0(R)$.

p. 141, 3.6.15(a) The a-invariant is $-\sum_{i=1}^n a_i$.

p. 142, l. −10 If M is *Artinian

p. 143, 3.6.18, l. 4 $^*H^i_{\mathfrak{m}}(M) \cong H_{\mathfrak{m}R_{\mathfrak{m}}}(M_{\mathfrak{m}})$

p. 143, 3.6.18 In the last line of the remark the local cohomology module should be denoted by $H^i_{\mathfrak{m}R_{\mathfrak{m}}}(M_{\mathfrak{m}})$.

p. 130 In lines 2–3 we require $\boldsymbol{x}$ to be a system of parameters. However, for the construction of the complexes $C^\bullet$ it is only necessary that $\boldsymbol{x}$ generates an $\mathfrak{m}$-primary ideal, and later on we use this fact several times.

p. 146, l. −15 $\operatorname{proj\,dim} R/I = 4$.

p. 146, l. −12 Remove the (repeated) sentence "The next case of interest is Gorenstein ideals of grade 4."

p. 148, 4.1.3 It is better to suppose $M \neq 0$ in Hilbert's theorem. Otherwise the zero polynomial must be allowed to be of degree -2.

p. 153 In Corollary 4.1.14(c) replace $\binom{n-d+i}{i}$ by $(n-d+i)!$.

p. 156 The k-algebras under consideration are finitely generated. (We have forgotten to require this condition explicitly.)

p. 157, l. −11 $I^\star = L(I)k[X_1, \dots, X_m]$

p. 158, l. 8 "…then $L(f) = \sum g_i L(f_i) \dots$": replace this by "$L(f) = gL(f_i)$ for some monomial g and some i"; replace $\sum g_i L(f_i)$ by gf_i in the following.

p. 160, ll. 12/13 …induction on d. For $d = 1$ …that $d > 1$ …

p. 161, l. −9 delete ", and $u \in S$ a monomial"

p. 162, l. 22 One can use 4.2.13 in order to derive the inequality $h(n) \leq \binom{n+m-1}{n}$.

p. 162, l. −5 Shorten the formula defining $a_{\langle d \rangle}$ by

$$a_{\langle d \rangle} = \binom{k(d)-1}{d} + \cdots + \binom{k(j)-1}{j},$$

where $j = \min\{i : k(i) \geq i\}$. (The version in the text is incorrect if $k(1) = 0$.) Furthermore add the definition $0_{\langle d \rangle} = 0$.

p. 166 In line 4 of the proof of 4.2.14 replace $\mathscr{M}_\backslash$ by $\mathscr{M}_n$. In the next line replace $I_n = 0$ by $J_n = 0$.

p. 168 In 4.2.17 one has to require that $\dim R > 0$.

p. 170, l. 6 Delete "be"

p. 171, l. 16 $H(R, j-2) - P_R(j-2) = -\dim_k {}^*H^1_{\mathfrak{m}}(R)_{j-2} < 0$

p. 176, l. 6 One need not require k to be algebraically closed.

p. 176, 4.4.4 degree of the Hilbert series

p. 185, 4.5.6 In the proof of part (b) it is claimed that $\operatorname{gr}_F(M)$ is a faithful $\operatorname{gr}_F(R)$-module (after the reduction to a faithful R-module M). This is not true in general, but one has $\operatorname{Supp}(\operatorname{gr}_F(M)) = \operatorname{Spec} \operatorname{gr}_F(R)$. and this property is sufficient for the continuation of the proof.

p. 200, 4.7.16 The ring in part (c) which is claimed to be non-Cohen-Macaulay is in fact Cohen-Macaulay. Replace it by $k[X^4, X^3Y, XY^3, Y^4]$.

p. 214 l. 3 $\bar{R} = R/xR$

p. 216 l. 19 only if $x < y$

p. 219, l. 9 $P_j(t) = 1 + t + \cdots + t^{r_j - 1}$

p. 228, *Shellings*, l. 3 $F_j \cap \bigcap_{i=1}^{j-1} F_i$

p. 228, paragraph below 5.2.14, l. 2 $P = \{x \in \mathbb{R}^d : \langle a_i, x\rangle \le 1,\ 1 \le i \le m\}$

p. 232, l. −4 as in Section 3.5

p. 248, l. 2 (11) → (8)

p. 261, l. 3 $\operatorname{Ker}\varphi \cap \mathbb{Z}C$ is contained in the normalization of C. Since C is positive, the normalization is positive as well, and one concludes that φ is injective.

p. 268 In the last part of the proof of 6.2.5 one only needs to prove that $H^i(L^\bullet \otimes M) = 0$ for $i > 0$ if M is an injective R-module. Furthermore the case $\mathfrak{p} = \mathfrak{m}$ has been forgotten, but it is evident that $H^i(L^\bullet \otimes E(R/\mathfrak{m})) = 0$ for $i > 0$. If $\mathfrak{p} \ne \mathfrak{m}$, then $H^i(L^\bullet \otimes E(R/\mathfrak{p})) = 0$ for all $i \ge 0$ as shown.

p. 269, 6.2.7(c) $\deg_{Y_1} f \ne 1$ and $\deg_{Y_2} f \ne 1$

p. 271, proof of 6.3.4(a) $X^z \in R_F$

p. 272 "If $z \in C$ ...": actually $\tilde{\mathscr{C}}(\mathscr{S})$ has not been defined for $\mathscr{S} = \emptyset$. So define it to be the zero complex.

The last index in the long exact sequence in (iii) should be $i - 1$.

p. 276, end of proof of 6.3.11 "The reader may prove this as an exercise ..." There is no need for an exercise or an external reference. In fact, the Hilbert polynomial of the canonical module can be read from Theorem 4.4.3 via graded local duality.

p. 280, proof of 6.4.2.(a) Delete the factor s_j from $s_j(u_{j1}a_1 + ... + u_{jn}a_n)$ in the second displayed formula.

p. 290, 6.4.15, l. 3 (a) $\Longrightarrow$ (b)

p. 290, 6.4.16.(b)(ii) $S = U \cap \mathbb{N}^n$

p. 291, 6.4.17, l. 1 $R = k[Y_1, Y_2, Z_1, Z_2]$

p. 293, 6.5.3.(c) ditto

pp. 295/296 Replace the last 3 lines of p. 295 and the first 2 lines of p. 296 by the following:

The elements cg_i^q, $i = 1, \dots, r+1$, are in the free $k[f_1, \dots, f_s]$-module F. Since $f_1, \dots, f_s$ form an F-sequence, there exist $h_{iq} \in F$ such that $cg_{r+1}^q = \sum_{i=1}^r f_i^q h_{iq}$. Applying the $k[f_1, \dots, f_s]$-homomorphism $\psi\colon S \to k[X_1, \dots, X_n]$ one has

$$c\psi(g_{r+1})^q = \sum_{i=1}^r f_i^q \psi(h_{iq}).$$

p. 296, l. 3 $\mu = X_1^{\mu_1} \cdots X_n^{\mu_n}$

p. 305, 7.1.8 In (a) the correct hypothesis for $H'A$ is that it is generated as a B-module by the standard monomials that contain a factor from H'.

Part (c) of the exercise is wrong in the generality stated, but it holds for ASLs.

pp. 308, 309 The correct Plücker relation is

$$[146][235] + [124][356] - [134][256] \\ + [126][345] - [136][245] - [123][456] = 0.$$

This leads to the representation

$$[146][235] = -[123][456] - [125][346] + [135][246].$$

p. 339, l. 1 *The reduction to the regular case.*

p. 350, l. −8, −4 depth $R_\mathfrak{p} \to$ depth $M_\mathfrak{p}$

p. 351, l. 5 ditto

p. 354, 9.1.15 Add the hypothesis that N is of finite projective dimension.

p. 354 In the proof of 9.2.1 replace R by S.

p. 360, proof of 9.3.2 We claim there is a minimal prime ideal $\mathfrak{p}$ such that height $\mathcal{O}(x) =$ height$((\mathcal{O}(x) + \mathfrak{p})/\mathfrak{p})$. In general, this is not true, as was pointed out to us by D. Eisenbud and B. Ulrich. (For unaccountable reasons we forgot to correct this mistake.)

Fortunately it is not difficult to solve the problem. First one passes to the completion $\hat{R}$ of R and replaces M by $M \otimes \hat{R}$. This causes no problems for height $\mathcal{O}(x)$ and big rank(M) since the extension $R \to \hat{R}$ is faithfully flat. Set $I = \mathcal{O}(x)$, and note that in view of what has to be proved it is enough to find a minimal prime ideal $\mathfrak{p}$ with height$(I + \mathfrak{p})/\mathfrak{p} \geq$ height I. We simply choose a prime ideal

$\mathfrak{p}$ with $\dim R = \dim R/\mathfrak{p}$. For all prime ideals $\mathfrak{q}$ of R with $\mathfrak{q} \supset I + \mathfrak{p}$ one then has

$$\operatorname{height} \mathfrak{q}/\mathfrak{p} = \dim R/\mathfrak{p} - \dim R/\mathfrak{q} \geq \dim R - \dim R/I \geq \operatorname{height} I,$$

as desired. The first equation uses that R is catenarian.

p. 364, l. 7 K-algebra $\to$ k-algebra

p. 364, l. −16 7.1 $\to$ I.7.1

p. 367, l. 6 $\chi(M, N) \to e(M, N)$

p. 380, 10.1.3(b), l. 4 only if $\operatorname{char} k \geq 7$

p. 384, l. −4 $a_1^q, \dots, a_j^q$ should also be multiplied by c.

p. 385, 10.1.12 the last item of the proposition should be labelled (e).

p. 394 The proof of 10.3.3(a) has a gap. (This gap exists already in [215].)

¿From 10.1.19 one concludes (as in [215]) that $cr^q \in J_{i-1}^{[q]} : x_i^q \subset (J_{i-1}^{[q]})^*$ for q large, but this is not sufficient for $r \in J_{i-1}^*$.

As C. Huneke suggested, one should prove 10.3.3(a) by the same method as 10.1.9 (and the best solution would be to derive 10.1.9 from it). We write $R = A/I$ as in 10.1.9, and choose $z_1, \dots, z_g, y_1, \dots, y_d$ according to 10.1.10. Then $z_1, \dots, z_g, y_1^q, \dots, y_d^q$ is an A-sequence for all $q = p^e$, $e > 0$.

Again we set $J = (z_1, \dots, z_g)$, and use the notation $\mathfrak{p}_1, \dots, \mathfrak{p}_m, \mathfrak{p}_{m+1}, \dots, \mathfrak{p}_n$ as in the proof of 10.1.9. There exist $d \in (\mathfrak{p}_{m+1} \cap \dots \cap \mathfrak{p}_n)^s \setminus (\mathfrak{p}_1 \cup \dots \cup \mathfrak{p}_m)$, s sufficiently large, such that $dI^s \subset J$ (the letters r and c are already in use). Choose a power $q' \geq s$ of p.

For q large we have a relation

$$c'(r')^q y_i^q - (a_1 y_1^q + \dots + a_{i-1} y_{i-1}^q) \in I$$

where c' and r' are preimages of c and r in A. Take the q'-th power with $q' \geq s$ and multiply by d. Then one obtains an equation

$$d(c')^{q'}(r')^{qq'} y_i^{qq'} - (a'_1 y_1^{qq'} + \dots + a'_{i-1} y_{i-1}^{qq'}) = b_1 z_1 + \dots + b_g z_g.$$

Using that $z_1, \dots, z_g, y_1^{qq'}, \dots, y_d^{qq'}$ is an A-sequence and taking residue classes modulo I yields

$$\tilde{d} c^{q'} r^{qq'} \in J_{i-1}^{[qq']}$$

for q large. Note that q' is *fixed*, and set $\tilde{c} = \tilde{d} c^{q'}$. Then $\tilde{c} \in R^\circ$, and $r \in (J_{i-1})^*$.

p. 395 In the proof of 10.3.5 one chooses t such that $(x_1^t, \dots, x_d^t) \subset (y_1, \dots, y_d)$ and writes $x_i^t = \sum_{j=1}^d a_{ij} y_j$

p. 395, 10.3.5 The proof of the proposition should be slightly reorganized, and the reference to 2.3.10 is not precise.

The existence of a tightly closed parameter ideal in the ring R implies that R is Cohen–Macaulay by 10.3.3(b) and 10.1.9. From now on we can use that all systems of parameters in R are R-sequences.

The reference to (the original version of) 2.3.10 does not cover the claim that $(y_1, \dots, y_d) = (x_1^t, \dots, x_d^t) : a$. However, the augmented version given above does so.

All other arguments can remain as they are.

p. 397, l. −19 $d \notin xR$

p. 400, l. −6, −5 $n \to d$

p. 402, l. 3 $(x_1^q, \dots, x_d^q) \to (x_2^q, \dots, x_d^q)$

p. 402, l. -6 $(x_1, \dots, x_d^q) \to (x_1^q, \dots, x_d^q)$

p. 403, l. −8 Replace $\mathcal{O}_X$ by $\mathcal{O}_W$ in the E_2^{pq} term.

p. 405, 10.3.28 $\max\{i : {}^*H_m^d(R)_i \neq 0\}$, $d = \dim R$

p. 405, l. 10.3.26 *Let k be*

p. 407, l. 1 insert comma between a_r and b_1

p. 422, [56] The title of the paper is ‘On multigraded resolutions’.

p. 422, [57] The article has appeared: *J. Pure Appl. Algebra* **122** (1997), 185–208.

p. 423, [74] The correct initial of Procesi is C.

p. 424, 88 J. A. Eagon and V. Reiner. Resolutions of Stanley-Reisner rings and Alexander duality. J. Pure Appl. Algebra 130 (1998), 265–275 (1998)

p. 427, 149 N. Hara. A characterization of rational singularities in terms of injectivity of Frobenius maps. Am. J. Math. 120 (1998), 981–996.

p. 431, 234 T. Kawasaki. On Macaulayfication of Noetherian schemes. – Appendix A: d^+-sequences. Appendix B: An example. Trans. Am. Math. Soc. 352 (2000), No.6, 2517–2552; appendix A: 2541–2547; appendix B: 2548–2552.

p. 438, 391 N. Hara and K. Watanabe. F-regular and F-pure rings vs. log terminal and log canonical singularities. J. Algebr. Geom. 11 (2002), 363–392.