Learning biases may prevent lexicalization of pragmatic inferences: a case study combining iterated (Bayesian) learning and functional selection

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Abstract

Natural languages exhibit properties that are difficult to explain from a purely functional perspective. One of these is the systematic lack of upper-bounds in the literal meaning of scalar expressions. This investigation addresses the development and selection of such semantics from a space of possible alternatives. To do so we put forward a model that integrates Bayesian learning in the replicator-mutator dynamics commonly used in evolutionary game theory. We argue that this synthesis provides a suitable and general model to analyze the dynamics involved in the use and transmission of language. Our results shed light on the semantics-pragmatics divide and show how a learning bias in tandem with functional pressure may prevent the lexicalization of pragmatic inferences.

Keywords: semantics; pragmatics; iterated learning; evolutionary game theory; scalar expressions

Introduction

Why are natural languages structured the way they are and not differently? In particular, what factors are involved in the selection of their semantic structure from a space of alternatives? A number of recent studies have began to investigate such issues pertaining to the development and emergence of linguistic features (see Steels 2015 and Tamariz & Kirby 2016 for recent overviews). While different methodologies have been put forward to this end, the overarching account is one of competing pressures: natural languages need to enable successful communication, as well as be suited for acquisition. That is, languages needs to be well-adapted to the communicative needs of its users but also need to be learnable to survive faithful transmission across generations.

The present investigation focuses on the interplay of such selective forces by means of a case study on the lexicalization of semantic upper-bounds. Using evolutionary game theory, framed in terms of language use and learning across generations, we investigate the prevalence of a lack of upper-bounds in the literal meaning of scalar expressions. The innovation of the model lies in its combination of functional pressure on successful communication, effects of learning biases on (iterated) Bayesian language learning (Griffiths & Kalish, 2007), and probabilistic models of language use in a population with distinct lexica (Frank & Goodman, 2012; Franke & Jäger, 2014; Bergen et al., to appear). Our results show that a learning bias for simple semantic representations coupled with communal language exposure sheds light on the prevalence of this feature provided bounds can be inferred via mutual reasoning. In turn, this gives an explanation to why certain pragmatic inferences fail to be lexicalized.

Conveying and lexicalizing upper-bounds

A considerably large class of natural language expressions do not lexicalize an upper-bound. That is, they are truth-conditionally compatible with more informative or "stronger" alternatives. Here, informativity is understood as an order over entailment. Examples in English include, among many others, numerals such as *five* and *six*, scalar adjectives like *cold* and *big*, as well as quantifiers like *some* and *many*. Their commonality is that their literal meaning is compatible with the truth of stronger (relevant) alternatives. For example, if it is true that 'Bill read five books' it may well be true that 'Bill read six/seven/... books'. Crucially, the latter entails the former. Analogously, if it is true that 'some students came to class' then this would also hold if it were true that 'all students came to class'. Prima facie this pattern may seem surprising. If it were true that 'some (but not all) students came', then it would putatively serve interlocutors better if *some* semantically ruled out the stronger *all* case. In practice, however, what is communicated often goes beyond literal meaning. In particular, mutual reasoning between interlocutors can lead to the enrichment of semantic content (Grice, 1975). Such *pragmatic reasoning* is driven by interlocutors' mutual expectations of rational language use. In this case the use of a less informative expression when a more informative one could have been used can license a defeasible inference that the stronger alternative does not hold (cf. Horn 1972; Gazdar 1979). In this way, a hearer who assumes the speaker to be knowledgeable and cooperative, i.e. able and willing to supply all relevant information at her disposition, can infer *some* to be pragmatically strengthened to convey an upper-bounded interpretation. This kind of reasoning was already hinted at above: such a speaker would have used the more informative alternative if it were true. Since she did not, a hearer can infer that it does not hold. Conversely, a speaker who reasons about the hearer can rely on her drawing this inference. Notwithstanding, while pragmatic reasoning provides an account of how interlocutors may derive and convey upper-bounds, the question why they are not part of the literal meaning still stands. More generally, while a divide between semantics and pragmatics is commonly agreed upon by linguists, under consideration of purely functional pressures this raises the challenge of providing justifications for the former's structure.

We see two main explanations for a lack of semantic upper-bounds in scalar expressions. The first is that pragmatic reasoning offers a general mechanism to strengthen the meaning of a wide range of expressions when the conditions outlined above hold. Crucially, cases where cooperativity or knowledge are not likely to be met are non-committal to whether stronger alternatives hold. If for all the speaker knows 'some students came' but she does not know whether 'all came', then the compatibility of *some* with (possibly) *all* succinctly conveys the speaker's uncertainty about the latter. Given that scalar expressions occur in contexts in which their upper-bounded reading is absent, one could argue for purely functional advantages of a lack of semantic upper-bounds: If expressing such a state of affairs is relevant and contextual cues provide enough information for a hearer to discern when a bound is conveyed pragmatically, then doing so is preferred over enforcing the bound overtly through a longer (more complex) expression, e.g. by stating 'some but not all'. That is, all else being equal, speakers prefer to communicate as economically as possible, and pragmatic reasoning enables them to do so. Further, this can be contrasted with the hypothetical alternative of lexicalizing two expressions – one with and one lacking an upper-bound. Four conditions may pressure language to English-like semantics over this alternative: (i) contextual cues are very reliable, morphosyntactic disambiguation is either (ii) not frequently necessary or (iii) not very costly, or (iv) having larger lexica is more costly than morphosyntactic disambiguation. In a nutshell (i) and (ii) place a heavy burden on the ability to retrieve contextual cues to a degree that is unlikely to undercut the benefit of safe communication with more expressions. As for (iii) and (iv), these seem mostly like technical solutions without a proper empirical basis.

A second explanation targets the contrast of the underlying semantic representation of upper-bounds and a lack thereof, positing a learning bias for the latter. That is, by virtue of their simpler representations, expressions lacking upper-bounds are more readily learnable than their bounded counterparts. If a bound can be supplied pragmatically this difference may explain the prevalence of such semantics. In the following we explore this hypothesis. While we do not want to argue that functionalist pressure may not play a role, we see a clear benefit in exploring whether learnability would not give us additional leverage. It should also be stressed that these explanations may well be complementary. A full-fledged account may reasonably be expected to involve an interplay between them.

Communication and selective dynamics

We employ evolutionary game theory to investigate the selection dynamics of language fragments that capture the contrast between upper-bounded meanings and their unbounded counterparts. Evolutionary game theory offers general and precise means to model the dynamics of linguistic pressures (Nowak & Krakauer, 1999; Huttegger & Zollman, 2013). More concretely, our model uses the replicator-mutator dynamics (see Hofbauer & Sigmund 2003 for an overview). In contrast to previous models we integrate (iterated) Bayesian learning in the dynamics (Griffiths & Kalish, 2007), as well as probabilistic language users of various pragmatic sophistication (Frank & Goodman, 2012; Franke & Jäger, 2014) together with multiple lexica (Bergen et al., to appear). In this way the model synthesizes core insights of previous proposals of language use and learning.

Linguistic interactions are represented by signaling games (Lewis, 1969). An interaction involves two players; a speaker and a hearer. The speaker has some private information about the state of the world and tries to convey this information to the hearer by sending a message. The hearer receives the message and interprets it. Communication is successful if the message's interpretation matches the speaker's information, i.e. when a message is interpreted correctly. We restrict our attention to games with two states, $S = \{s_1, s_2\}$, and two messages, $M = \{m_1, m_2\}$. This allows for a minimal contrast between weaker (non-)upper-bounded expressions and stronger alternatives in states were a bound is to be conveyed. Players base their choices on lexica that specify the semantics of their language. Formally, a lexicon L is a Boolean $(|S|, |M|)$-matrix such as L_a and L_b below.

$$L_a = \begin{array}{c c} & \begin{array}{cc} m_1 & m_2 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \end{array} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array} \qquad L_b = \begin{array}{c c} & \begin{array}{cc} m_1 & m_2 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \end{array} & \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \end{array}$$

According to L_a, message m_1 is true in s_2 and false in s_1. The converse holds for m_2. In L_b, however, m_1 is true in both s_1 and s_2. In this way L_a stands for a language that encodes an upper-bound for m_1, e.g. *some but not all*, whereas L_b represents one where m_1 lacks such a bound, e.g. *some*.

To contrast literal language use to its pragmatic counterpart we consider two kinds of players. *Literal players* communicate according to the semantics of their lexica. *Pragmatic players* reason about their interlocutors and base their choices on this reasoning. In other words, pragmatic speakers take into account how listeners would interpret a message. In turn, pragmatic listeners take the speaker into account when interpreting. This kind of signaling behavior shares the core features common to models of rational language use such as Rational Speech Act models (Frank & Goodman, 2012) and Quantal- and Best-Response models (Franke, 2009; Franke &

Jäger, 2014). Following this line of research, player behavior can be captured by a hierarchy of reasoning types. Literal players constitute the bottom of the hierarchy, level 0, being solely guided by the semantics of their language. Players of level $n+1$ behave rationally according to level n behavior of their interlocutors. Presently it suffices to consider players of no order higher than 1 as the cases considered here offer little room for further pragmatic refinement. Literal hearer and speaker behavior are summarized in (1) and (2), and their pragmatic counterparts in (3) and (4).

$$R_0(s|m;L) \propto P^*(s)L_{sm} \quad (1)$$

$$S_0(m|s;L) \propto \exp(\lambda L_{sm}) \quad (2)$$

$$R_1(s|m;L) \propto P^*(s)S_0(m|s;L) \quad (3)$$

$$S_1(m|s;L) \propto \exp(\lambda R_0(s|m;L)^\alpha) \quad (4)$$

Speakers strive to maximize their communicative success but may occasionally make mistakes in computing their expected utility. This is regulated by the soft-max parameter λ, $\lambda > 0$ (Luce, 1959; Sutton & Barto, 1998). Intuitively, the larger λ the more faithful a speaker's choices are to the maximization of expected communicative success. A second parameter α, $\alpha \in [0,1]$, controls the tension between semantics and pragmatics. Lower α-values lead to more literal signaling whereas larger values lead to stronger pragmatic behavior. $P^* \in \Delta(S)$ is a prior over states. For simplicity in the following we assume this prior to be common and uniform. Lastly, a *player type* is a combination of signaling behavior, i.e. either literal or pragmatic, and a lexicon.

With these components at hand population-level dynamics can be analyzed. As noted above two key components are involved: communicative fitness and lexicon learnability. The fitness of a type i is given by its expected utility. That is, the fitness of type i, f_i, in a population x is the sum of its weighted expected utility, $f_i = \sum_j x_j U(x_i,x_j)$, where x_j is the proportion of players of type j in x and $U(x_i,x_j)$ is the symmetrized expected utility of x_i and x_j.[1] A type's fitness indicates how well it communicates within a population. The average fitness of the population is $\Phi = \sum_i x_i f_i$.

The second component is given by a learning matrix Q, which adds (iterated) Bayesian learning to the dynamics (Griffiths & Kalish, 2007). It operationalizes the assumption that new generations of learners combine prior learning biases with observations over language use. This determines the probability of adopting a type. As a consequence, lexica that better explain observations are more likely to be passed onto the next generation faithfully. Here we assume that the adoption of a lexicon is not solely dependent on how well a parent's lexicon explains the data but also on its proportional representation in the population. The intuition is that it may be possible that a lexicon that is – in principle – well-suited for acquisition may nevertheless not be adopted when only few types of the preceding population used it. More concretely, Q_{ji} specifies the probability that a child of type j will be of type i. This is weighted by probability of being exposed to type i, $N_{ji} = x'_i Q_{ji}$ where x'_i is proportion of i in the previous generation. Q itself depends on observations of linguistic utterances and a prior. The set of possible observations is given by all combinations between state-message pairs, $O = \{\langle\langle s_1,m_i\rangle,\langle s_2,m_j\rangle\rangle | m_i,m_j \in M\}$. A member of O encodes that a teacher produced m_i in state s_1 and m_j in s_2, i.e. it encodes one witnessed message for each state. A datum d is a sequence of length k of members of O. Learners witness such data sequences. Accordingly, $Q_{ji} \propto \sum_d P(d|t_j)P(t_i|d)$ with $P(t_i|d) \propto P(t_i)P(d|t_i)$. The prior $P(t)$ gives the probability of a type without witnessing data, i.e. it encodes the learning bias of players prior to linguistic exposure. The likelihood $P(d|t)$ gives the probability of data being produced by a type. The posterior thusly yields a combination of prior expectations and data that captures a type's fit to the data. This, in turn, regulates the transmission of its lexicon. Putting these components together, the discrete selection dynamics are captured by $\hat{x}_i = \sum_j N_{ji} \frac{x_j f_j}{\Phi}$ (cf. Hofbauer & Sigmund 2003).

As mentioned above the prior captures the learning bias of players. For simplicity we assume it to be solely dependent on lexica and not on a type's behavior. In particular it biases learners for simpler semantic representations over more complex ones. This encodes the intuition that the semantic representation of an upper-bound is more complex than a lack thereof. Given that semantics are only implicitly represented through a lexicon's Boolean matrix, the bias is regulated by a parameter c that operates over cells of a lexicon.[2] As illustrated above by lexica L_a and L_b we let s_1 stand for a "some but not all"-state and s_2 for an "all"-state. Accordingly, the prior biases players against lexica in which a message m holds true only of the former and not the latter, as in L_a. The literal meaning of English *some* corresponds to a message true of s_1 and s_2 in such a fragment. All other semantics are a priori equally probable. Thus, $P(t_i) \propto n - c \cdot r$ where n is the number of states in S and r that of upper-bounded messages only true of s_1 in t_i's lexicon.

Analysis

We consider a total of 12 types by pairing six lexica with literal and pragmatic players. The lexica are specified in Table 1. L_1 to L_3 are not optimal for communication because they

[1] $U(x_i,x_j) = [U_S(x_i,x_j) + U_R(x_i,x_j)]/2$. $U_S(x_i,x_j)$ and $U_R(x_i,x_j)$, the expected utility of speaker type x_i and hearer type x_j and vice versa, are respectively $\sum_s P^*(s) \sum_m S_n(m|s;L) \sum_{s'} R_o(s'|m;L)\delta(s,s')$ and $U_S(x_j,x_i)$ for n and o being the reasoning level of i and j, and $\delta(s,s') = 1$ iff $s = s'$ and 0 otherwise.

[2] In principle this contrast could be made more precise with an adequate representational language, e.g. by measures over representational complexity. There is a growing effort to develop such empirically testable representational languages. For instance, the so-called *language of thought* has been put to test in various rational probabilistic models that show encouraging results (see e.g. Katz et al. 2008; Piantadosi et al. under review, 2012 and references therein). We think that our assumption is well-warranted as a working hypothesis and decide against such an enrichment given that the introduction of a larger framework would also require further assumptions and justifications.

$$L_1 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \quad L_2 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad L_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$L_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad L_5 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \quad L_6 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Table 1: Space of possible lexicon fragments considered.

assign the same meaning to all their messages. They were included to showcase the selection process for larger ranges of lexica. L_4 and L_5 are the crucial lexica that represent upper-bounded semantics and a lack of it, respectively. L_6 is similar to L_5 in that two messages are true of the same state but differs from it in assigning upper-bounded semantics to m_2. In the following we focus our analysis on the contrast between literal and pragmatic types using lexica L_4 and L_5. Note in particular that the linguistic behavior of pragmatic L_4 and L_5 can be close to indistinguishable as both are able to convey upper-bounds. The former semantically, the latter pragmatically. Depending on the parameters, however, there might be slight differences between the probability with which speakers would (erroneously) use a semantically false description and that they would (erroneously) use a pragmatically suboptimal description. This contrasts with literal L_5 which lacks the means to convey an upper-bound by m_2. Differences between pragmatic L_4 and L_5 are expected to mainly depend on the learning bias. Things are less clear for literal L_5 contrasted with literal/pragmatic L_4 as the former has a learning advantage but is expected to fare worse in terms of communicative fitness in virtue of ambiguous m_2.

The learning matrix Q was computed with data sequences of length 20. Pilot simulations showed that changes in sequence length influence the population in a predictable way: smaller values lead to more heterogeneous populations whereas larger ones lead to more pronounced differences. This is expected insofar as the likelihood that a sequence of length 1 was produced by any type is relatively uniform (modulo prior) whereas the likelihood of types with L_1 - L_3 to produce, for instance, a sequence of 10 observations consistently with the same state-message combination is less likely than for pragmatic players using L_4 - L_6 or literal L_4. Thus, while noteworthy, the sequence length has no direct bearing on our main contrast of interest. Similar considerations hold for α and λ – set to 1 and 50 in the following. Overall, lower rationality λ or more pragmatic violations in α lead to a higher selection of lexica with semantic upper-bounds. The fitness of pragmatic behavior increases with higher λ-/α-values. In other words, they level the functional contrast between L_4 and L_5. Due to space restrictions we leave a more detailed analysis of their interaction to future research and focus on the learning bias instead.

All simulations were initialized with an arbitrary distribution over types constituting the population's first generation, and were run for 20 generations. As with the length of data sequences this corresponded to a developmental plateau after which no change was registered.

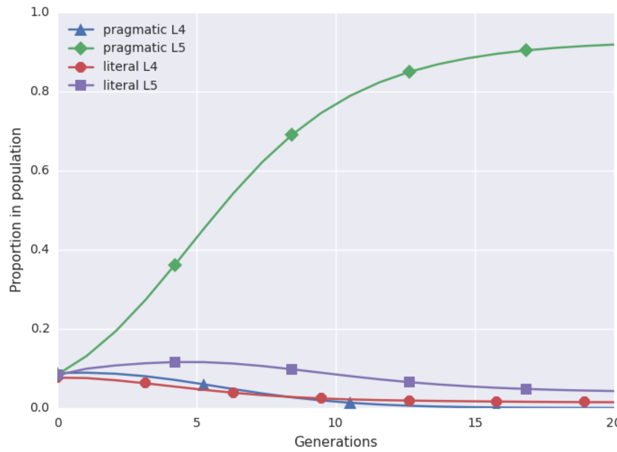

Figure 1: Mean population development of target types across 20 generations of 1000 populations ($\alpha = 1, \lambda = 50, k = 20, c = 0.85$).

	0	.01	0.03	0.05	0.07	0.09	0.85
lit. L_1	.001	.002	.002	.004	.004	.002	.004
lit. L_2	.001	.002	.001	.001	.001	0	0
lit. L_3	.001	.003	.001	.002	.002	.001	.004
lit. L_4	.193	.193	.165	.177	.187	.142	.022
lit. L_5	.022	.022	.039	.04	.033	.049	.042
lit. L_6	.023	.023	.028	.027	.028	.025	.002
prag. L_1	.001	.001	.001	.002	.004	.003	.005
prag. L_2	.001	.001	.001	.001	.001	0	0
prag. L_3	.001	.002	.002	.002	.004	.002	.006
prag. L_4	.257	.234	.23	.209	.205	.208	0
prag. L_5	.249	.26	.29	.328	.329	.379	.914
prag. L_6	.25	.237	.24	.207	.202	.188	0

Table 2: Mean proportion of types after 20 generations with different learning biases in 1000 populations ($\alpha = 1, \lambda = 50, k = 20$).

Results. We report the mean results obtained in 1000 simulations for each set of parameters. Figure 1 illustrates the development of the target types over 20 generations with $c = 0.85$, i.e. a strong bias against semantic complexity. Figure 2 shows the effect of the learning bias after 20 generations for $c \in [0, 100]$. More detailed results for all types across a sample of c-values are presented in Table 2.

In sum, the results show that in the present setup a weak bias is sufficient to lead to a selection of L_5 over L_4. This effect increases with the bias' strength provided L_5 users are pragmatic. That is, learning biases may prevent the lexicalization of pragmatic inferences and explain the prevalence of L_5-like semantics. Note further that the literal L_5 type is underrepresented across all values of c. This highlights that while the bias plays a major role for the contrast between L_4

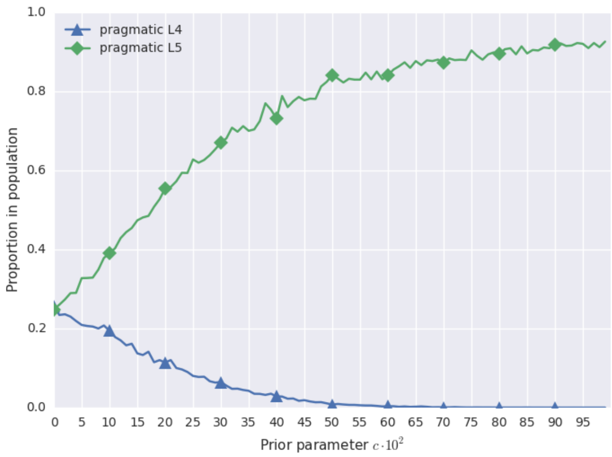

Figure 2: Mean proportion of pragmatic L_4 and L_5 types after 20 generations with $c \in [0, 100]$ in 1000 populations ($\alpha = 1, \lambda = 50, k = 20$).

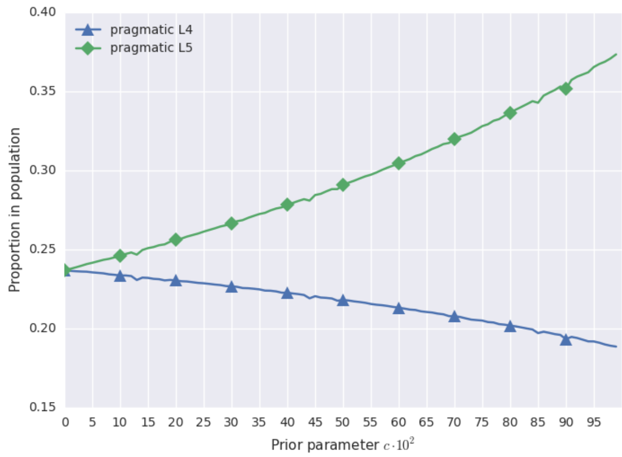

Figure 3: Proportion of pragmatic L_4 and L_5 types after 20 generations of parental teaching with $c \in [0, 100]$ ($\alpha = 1, \lambda = 50, k = 20$).

and L_5, in itself it does not enable types that fail to convey an upper-bound to establish themselves in the population. This suggests pragmatic reasoning to be paramount to the selection of L_5 semantics.

Discussion

Broadly speaking these results confirm our expectations that a lack of semantic upper-bounds coupled with pragmatic reasoning can overcome selective pressures and stabilize in a population provided there is a bias for simpler representations. This outcome is particularly encouraging in light of the other potential advantages of a lack of semantic upper-bounds discussed above. The model gives a justification for lexicalization patterns found in natural language, as well as the failure to lexicalize certain pragmatic inferences. That is, while the puzzle raised by semantics is hard to explain by purely functional means, we suggest part of the answer to lie in learnability. Simpler semantic representations are more likely to be learned, and pragmatic reasoning can counteract functional disadvantages that may otherwise be incurred. This result is of particular relevance for the longstanding assumption of a divide and interaction between these two linguistic modules and offers an account of why (certain) pragmatic inferences are not part of the literal meaning of expressions. It furthermore leaves open the possibility of such inferences to fossilize when they do not compete against a lexical simplicity bias.

A virtue of this model is that it allows for analysis specific modifications and extensions. Straightforward extensions include larger hypothesis spaces, and larger or different lexicon fragments. In the case of scalar expressions we tacitly assumed pragmatic reasoning to come at no cost. However, there is experimental evidence for the assumption that the pragmatic derivation of upper-bounds costs effort and takes additional processing time (cf. Neys & Schaeken 2007; Huang & Snedeker 2009). This raises the interesting question at which point such usage-based cost undercuts the learnability advantage of simpler semantic representations.

Lastly, a crucial component of this model is the learning matrix Q together with its weighted variant N. The latter describes "communal teaching" where learning a lexicon is influenced by its proportional representation in the preceding generation. This diverges from the mean field dynamics of classical (dynamic) evolutionary game theory and iterated learning. Replacing the weighted matrix N by Q, i.e. not weighting the learning matrix, allows for an inspection of the model's predicted outcome for a more classical case of "parental teaching". Figure 3 shows the effect of the learning bias for this case (cf. Figure 2).

Note first that the main result of the prior driving selection of L_5 over L_4 holds here as well. However, the resulting populations are more heterogeneous than in the communal setting. The intuition behind communal learning was that learning should be influenced by the proportion in which languages are used in a population. However, a stage of exposure to communal language use is currently not represented in the model but only implicit in the weights applied to the Bayesian learning matrix. This step may be regarded as conceptually problematic given that it assumes "implicit exposure" to the true distribution of types in the population. Notwithstanding, we see it as a first step in the right direction to capture what constitutes an important factor in learning pressures. Further, it retains but strengthens the results obtained from a more standard learning setup. It is nevertheless important to highlight the weaker results obtained from parental learning as they suggest a missing piece to our overall account. Ideally one would expect the dynamics to favor a well-adapted lexicon over time even when only learning from parents. This would yield a stronger account of the advantage of a lexicon without a population's involvement in the learning process. It is possible that a more complex setup where the speaker wants to convey not only bounded but also upper-unbounded states may shed light on this issue, as discussed in §2. Alternatively, a similarity bias over lexica may enable for a gradual adoption of particular languages. A more involved analysis and comparison between static learning matrices and weighted variants, as well as an explicit representation of exposure to linguistic data from the population raise many interesting technical and conceptual challenges we hope to be addressed in future research.

Conclusion

The development of natural languages is driven by complex intertwined pressures. Drawing from past insights we put forward a model that allows for a general and malleable integration of core aspects involved in this process. Chiefly, the model combines functional pressure, iterated Bayesian learning, and probabilistic speaker and hearer models. In particular, this analysis addresses longstanding issues concerning the semantics-pragmatics divide. It shows that when pressured for learnability and expressivity, the former force drives for simpler semantic representations inasmuch as pragmatics can compensate for them in language use. As a result semantic patterns can be explained in virtue of the linguistic behavior of their users and their complexity. Furthermore, they give an answer to why natural languages do not lexicalize certain pragmatic inferences.

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