List of Symbols

$\vee$	join in lattice, 72
$\wedge$	meet in lattice (or exterior product), 72
$<_{\text{deglex}}$	degree lexicographic order, 3
$<_{\text{lex}}$	lexicographic order, 3
$<_{\text{revlex}}$	reverse lexicographic order, 3
$[-]$	cohomological shift, 370
$- \boxtimes -$	external tensor product, 328
$(-)^G_\chi$	eigenspace corresponding to the character χ of G, 356
$\bullet$	dot action of the symmetric group $\mathfrak{S}_n$ on $\mathbb{Z}^n$, 416
1_G	unit element for an algebraic group G, 350
$[\ldots \mid \ldots] \preceq [\ldots \mid \ldots]$	partial order of minors, 72
$[a_1 \ldots a_t \mid b_1 \ldots b_t]$	minor with rows $a_1, \ldots, a_t$ and columns $b_1, \ldots, b_t$, 70
$[a_1 \ldots a_t \mid b_1 \ldots b_t]_X$	minor $[a_1 \ldots a_t \mid b_1 \ldots b_t]$ of matrix X, 70
$\langle a_1 \ldots a_t \mid b_1 \ldots b_t \rangle$	diagonal of minor $[a_1 \ldots a_t \mid b_1 \ldots b_t]$, 80
$[a_1 \ldots a_m]$	maximal minor with columns $a_1, \ldots, a_m$, 70
$\mathbb{A}_{\mathbf{B}}(\mathcal{E})$	affine bundle, 329
$a_i(M)$	highest degree in $H^i_{Q_R}(M)$, 276
$\alpha_k(\sigma)$	$\sum_{i \le k} s_i$ for $\sigma = (s_1, \ldots, s_u)$, 78
$\alpha_k(\Sigma)$	$\alpha_k(\lvert\Sigma\rvert)$, 93
α_k^*	dual of α_k, 119
$\widehat{\alpha}_k(r)$	maximum of α_k taken over inc-decompositions of r, 118
$\widehat{\alpha}_k^*$	dual of $\widehat{\alpha}_k$, 119
Ann	annihilator of a module, 381
$a(R)$	a-invariant of graded algebra R, 136
$\operatorname{ara}(I)$	arithmetic rank of an ideal I, 467
$\mathcal{A}_{r,s},\ \mathcal{B}_{r,s}$	sets used to describe syzygies of determinantal ideals, 472
$\mathfrak{A}(s)$	weights for simple $\mathcal{D}$-module supported on rank s matrices, 465

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$\mathfrak{a}^t$	shorthand for the pair $(\mathfrak{a}, t)$, 265
A_t	algebra generated by t-minors, 195
$\beta_{ij}(M)$	graded Betti number of module M, 38
Bigheight I	maximum height of an associated prime ideal of I, 44
bigheight I	maximum height of a minimal prime ideal of I, 44
$B_{i,j}(I)$	syzygies of the ideal I, 470
$B_I(q)$	equivariant Betti polynomial for the ideal I, 471
$\mathrm{cd}_S(I)$	cohomological dimension of an ideal $I \subset S$, 467
$\mathrm{cl}(I)$	divisor class of ideal I
$\mathrm{Cl}(R)$	divisor class group of ring R
$C(M)$	cone generated by monoid M, 173
Coker	cokernel of a homomorphism
`computel`	function to find value of l in Lemma 10.2.5, 389
$\mathcal{D}$	product of minors along diagonals, 193
$\mathcal{D}_t$	product of minors of size $\geq t$ along diagonals, 241
$\boldsymbol{d}_{uv}$	special diagonal matrix, 209
$D^d\mathcal{E}$	divided power of locally free sheaf, 327
$D^d V$	divided power of a free module, 325
deg	degree
$\mathrm{Deg}_M(Z_1, \ldots, Z_n)$	multidegree of M, 147
$\langle\Delta\rangle$	product of diagonals of bitableau Δ, 80
$\langle\delta\rangle$	diagonal of minor δ, 80
$\lvert\Delta\rvert$	shape of bitableau Δ, 71
$\lvert\delta\rvert$	size of minor δ, 71
$\Delta^{\{r\}}$	rth secant of simplicial complex Δ, 142
$\Delta_1 * \cdots * \Delta_r$	join of simplicial complexes $\Delta_1, \ldots, \Delta_r$, 142
Δ_G	comultiplication on $K[G]$, 351
$\Delta(I)$	simplicial complex defined by squarefree monomial ideal I, 48
Δ_M	comodule map for M, 356
depth$M$	depth of module M, 38
det$V$	determinant of a free module, 326
$\mathbf{D}_n$	algebraic group of diagonal matrices (group scheme), 354
D_{r+1}	initial algebra of R_{r+1}, 219
D^-	derived category of bounded above complexes, 369
D^+	derived category of bounded below complexes, 369
$e(M)$	multiplicity of module M, 25
$\mathcal{E}_b$	fiber of a sheaf $\mathcal{E}$ at the point b, 329
$e_b(M)$	mixed multiplicity of module M, 147
$\mathcal{E}^\vee$	dual of locally free sheaf, 328
$\mathcal{E}^j_{\sigma,l}(I)$	GL-module isomorphic to $\mathrm{Ext}^j_S(J_{\sigma,l}, S)$, 456
ε	weight defined on shapes, 212
ε_G	counit on $K[G]$, 351
$\mathcal{E}xt$	sheaf Ext, 373

F	(often) Frobenius map, 230
$\langle F_1, \ldots, F_p \rangle$	simplicial complex generated by faces $F_1, \ldots, F_p$, 110
$\mathbb{F}_{\mathbf{B}}(\mathcal{E})$	complete flag bundle, 332
$\mathbb{F}_{\mathbf{B}}(L; \mathcal{E})$	partial flag bundle, 331
$\mathsf{F}(\Delta)$	set of facets of simplicial complex Δ, 109
F^e	iterated Frobenius map, 243
$f^!$	exceptional inverse image functor, 370
$f^! K$	dualizing complex, 370
$f \overline{\otimes} g$	tensor product followed by counit, 351
$\mathcal{F}(\mathfrak{p})$	face of cone of weights defined by $\mathfrak{p}$, 214
f_*	direct image functor
$F_* R$	R-module structure on R defined by Frobenius map, 230
$F^e_* R$	R-module structure on R defined by F^e, 243
f^*	inverse image functor
F_t	set of facets of Π_t, 188
$\mathrm{fpt}_R(\mathfrak{a})$	F-pure threshold of ideal $\mathfrak{a}$, 265
$f \star \mathrm{Tr}$	the map $F_* R \to R$ defined by $g \mapsto \mathrm{Tr}(g)$, 236
$\mathbf{G}_a$	additive group, 351
γ_t	valuation defined by prime ideal of t-minors, 88
$\gamma_t(s_1, \ldots, s_u)$	$\gamma_t(\Delta)$ for a bitableau of shape $(s_1, \ldots, s_u)$, 89
$\widehat{\gamma_t}(r)$	maximum of γ_t taken over inc-decompositions, 121
$\Gamma(U, -)$	functor of sections on an open set U
$\mathbb{G}_{\mathbf{B}}(l; \mathcal{E})$	Grassmann bundle, 329
`generateSigma0`	function to generate minimal partitions relative to $\subset$, 388
$\mathbb{G}$	$\mathrm{GL}(m, K) \times \mathrm{GL}(n, K)$, 95
$\mathrm{GL}(m, K)$	group of invertible $m \times m$ matrices, 95
$\mathfrak{gl}(m\|n)$	general linear Lie superalgebra, 476
$\mathbf{GL}_n$	general linear group (group scheme), 352
$\mathbf{G}_m$	multiplicative group, 351
$\mathrm{gp}(M)$	group of differences of monoid M, 173
$\mathrm{gr}_{\mathcal{F}}(R)$	associated graded ring of filtration $\mathcal{F}$, 53
$\mathrm{gr}_I(R)$	associated graded ring of ideal I, 186
$\mathrm{grade}(I, M)$	grade of ideal I with respect to module M, 276
$H^i(\mathbf{B}, -)$	sheaf cohomology functor on a variety $\mathbf{B}$
H_i^+	linear halfspace, 173
$H(Q, M)$	Koszul homology of module M with respect to ideal Q, 278
$H_I^j(-)$	functor of local cohomology with support in an ideal I
$\hom_w(f)$	homogenization of f w.r.t. weight vector w, 32
$\hom_w(V)$	homogenization of vector space V w.r.t. weight vector w, 32
$\mathrm{Hom}_{\mathrm{GL}}$	homomorphisms of GL-modules, 447
$\mathscr{H}om$	sheaf of homomorphisms, 370

$h_{r\times s}(q)$	polynomial encoding linear strands in Lascoux' resolution, 472
$H_V(t)$	Hilbert series of V, 24
$H(V, \ )$	Hilbert function of V, 24
$h(\underline{y})$	height of the partition $\underline{y}$, 341
$\overline{I}_\Delta$	face or Stanley–Reisner ideal of simplicial complex Δ, 48
$I^{(d)}$	dth symbolic power of an ideal I
i_G	inverse for an algebraic group G, 350
$I * J$	join of ideals I and J, 138
$I_{\leq j}$	subideal of I generated by the elements of degree $\leq j$, 298
$\mathrm{In}(\sigma)$	initial tableau of shape σ, 79
$\mathrm{In}(V)$	set of initial monomials of elements of V, 22
$\mathrm{in}(A)$	initial (sub)algebra of (sub)algebra A, 14
$\mathrm{in}(f)$	initial monomial of f, 4
$\mathrm{in}(I)$	initial ideal of ideal I, 4
$\mathrm{in}(V)$	initial vector space of vector space V, 22
$\mathrm{in}_w(V)$	initial subspace (algebra, ideal) w.r.t. weight w, 27
$\mathrm{inic}(f)$	initial coefficient of f, 4
$\mathrm{init}(f)$	initial term of f, 4
$\mathrm{init}_w(f)$	initial term (or form) of f w.r.t. weight w, 26
$\mathrm{Ins}(r)$	standard tableau obtained from sequence r by insertion, 117
$I^{[p]}$	Frobenius power of ideal I, 234
$I^{[p^e]}$	iterated Frobenius power of ideal I, 243
$I^{\{r\}}$	rth secant ideal of I, 140
$I^{(\sigma)}, I^{(\sigma)}_{>}$	ideal generated by all bitableaux of shape $\geq \sigma$ and $> \sigma$ resp., 95
I^σ	product $I_{s_1} \cdots I_{s_u}$, 90
$I^{(\Sigma)}$	ideal defined by the shapes in Σ, 380
I_Σ	GL-invariant ideal associated to a set of shapes Σ, 451
I_σ	GL-invariant ideal associated to a shape σ, 449
$\mathcal{I}^{(\sigma)},\ \mathcal{I}^{(\Sigma)}$	sheaf of ideals defined by shape, 393
I^*	tight closure of ideal I, 245
$I_t^{(k)}$	kth symbolic power of I_t, 90
$I_t, I_t(X)$	ideal generated by t-minors (of matrix X), 70
$\mathcal{J}_l^{(\sigma)}$	quotient of sheaves of ideals defined by shape, 396
$J_l^{(\sigma)}$	quotient of ideals defined by shape, 380
$J_{\sigma,l}$	quotient of GL-invariant ideals, 453
$J_t^{\min}(m,n),\ J_t^{\min}$	$\mathbb{G}$-module representing minimal generators of $J_t(m,n)$, 202
$J_t(m,n),\ J_t$	defining ideal of $A_t(m,n)$, 199
$\kappa(b)$	residue field at the point b, 329

$K[\Delta]$	Stanley–Reisner ring of simplicial complex Δ, 48
Ker	kernel of a homomorphism
$K_{\lambda,\mu}$	Kostka number of partitions λ, μ, 151
$K[M]$	monoid algebra, 172
$K[\mathcal{M}_\tau]$	coordinate ring of flag variety, 79
λ^*	dual of shape λ, 119
Λ_t	map taking a linear map to its tth exterior power, 208
$\bigwedge^d \mathcal{E}$	exterior power of locally free sheaf, 328
$\bigwedge^d V$	exterior power of a free module (or vector space)
$\bigwedge^d V^\vee$	exterior power of $V^\vee$, also dual of the exterior power of V, 327
lcm	least common multiple
$\mathcal{L}_i^{\mathcal{E}}$	tautological line bundle, 332
L_σ, ${}_\sigma L$	subspaces generated by all right resp. left initial bitableaux, 95
$\ell(\sigma)$	length of a permutation σ, 416
$\mathfrak{m}_b$	maximal ideal of $\mathcal{O}_{\mathbf{B},b}$, 329
M^G	G-invariants of M, 356
m_G	multiplication for an algebraic group G, 350
$M^{(i,*)}$	ith homogeneous component of M w.r.t. $(1, 0)$-grading, 288
$M^{(*,j)}$	jth homogeneous component of M w.r.t. $(0, 1)$-grading, 288
$M_{\geq j}$	truncated module $\bigoplus_{i\geq j} M_i$, 283
$\mathrm{Mon}(R)$	set of monomials in R, 2
M_σ	irreducible $\mathbb{G}$-representation, 98
$\mathcal{M}_\sigma(\mathcal{F}_1, \mathcal{F}_2)$	a direct summand of $\mathrm{Sym}^d(\mathcal{F}_1 \otimes \mathcal{F}_2)$ (in characteristic zero), 435
$\mathcal{M}_\tau$	set of bitableaux generating coordinate ring of flag variety, 79
$\mathcal{M}_t(X)$	set of t-minors of X, 70
$\mu(M)$	minimal number of generators of module M, 225
$\mu_1 \leq \mu_2$	monomial μ_1 precedes μ_2 in monomial order, 3
$\mathrm{mult}_{(\sigma\|\tau)}$	multiplicity of bi-shape $(\sigma\|\tau)$ in $P_t(m, n)$, 201
$\mathrm{mult}_{(\sigma\|\tau)}(E)$	multiplicity of bi-shape $(\sigma\|\tau)$ occurring in E, 201
$\mathcal{M}(X)$	set of nonempty minors, 70
$\mathbb{N}$	set of nonnegative integers
$\mathbb{N}_{>0}$	set of positive integers
$\nu_e^I(\mathfrak{a})$	$\sup\{r \in \mathbb{N} : \mathfrak{a}^r(I^{[p^e]} : I) \not\subset \mathfrak{m}^{[p^e]}\}$, 265
$\mathcal{O}_{\mathbf{B}}$	structure sheaf of a variety $\mathbf{B}$
$\mathcal{O}_{\mathbf{B},b}$	local ring at the point b, 329
$\mathcal{O}^{\mathcal{E}}_{\mathbb{F}_{\mathbf{B}}(\mathcal{E})}(\underline{y})$	line bundle associated to the weight $\underline{y} \in \mathbb{Z}^n$, 332
ω_R, $\omega(R)$	(graded) canonical module of R, 136
$\omega_{\mathbb{F}_{\mathbf{B}}(\mathcal{E})/\mathbf{B}}$	relative canonical sheaf for a complete flag bundle, 333
$\omega_{\mathbb{G}_{\mathbf{B}}(l;\mathcal{E})/\mathbf{B}}$	relative canonical sheaf for a Grassmann bundle, 330

$\Omega_{\mathbb{G}_{\mathbf{B}}(l;\mathcal{E})/\mathbf{B}}$	sheaf of relative differentials for a Grassmann bundle, 330
$\Omega_{\mathbb{P}_{\mathbf{B}}(\mathcal{E})/\mathbf{B}}$	sheaf of relative differentials for a projective bundle, 330
$\omega_{X/Y}$	relative canonical sheaf
$\mathrm{Paths}(\mathcal{P}, \mathcal{Q})$	number of families of nonintersecting paths from $\mathcal{P}$ to $\mathcal{Q}$, 131
$\mathrm{Paths}(\mathcal{P}, \mathcal{Q}, z)$	generating function of families of paths from $\mathcal{P}$ to $\mathcal{Q}$, 134
$\mathbb{P}_{\mathbf{B}}(\mathcal{E})$	projective bundle, 329
$\mathcal{P}(d)$	set of partitions of d
$\mathfrak{p}_i$	$\mathbb{G}$-stable prime ideal in A_t, 212
Π_2	simplicial complex defined by initial ideal of I_2, 108
Π_t	simplicial complex defined by initial ideal of I_t, 128
π_i	valuation on A_t, 212
$\mathcal{P}_m$	set of shapes with parts of size $\leq m$, 382
$P \leq Q$	partial order of paths, 112
$\operatorname{proj\,dim} M$	projective dimension of module M
$\underline{\mathrm{Proj}}$	relative Proj, 329
$P_t(m, n), \overline{P_t}$	polynomial ring in indeterminates representing the t-minors, 198
Q	often ideal $(X_1, \ldots, X_n)$, 277
$Q_{(0,1)}$	ideal of $R^{(0,*)}$ generated by $R_{(0,1)}$, 287
$Q_{(1,0)}$	ideal of $R^{(*,0)}$ generated by $R_{(1,0)}$, 287
$\binom{u}{v}_q$	q-binomial coefficient, 465
$\mathfrak{q}_k$	$\mathbb{G}$-stable prime ideal in A_t, 212
$\mathcal{Q}_l^{\mathcal{E}}$	tautological rank l quotient sheaf, 330
Q_R	ideal generated by R_1, 276
$(R\|C)$	bitableau with row tableau R and column tableau C, 71
R_χ	semi-invariants of R for character χ, 220
R°	set of elements of R not in any minimal prime ideal, 243
$R(\Delta)$	set of relevant faces of simplicial complex Δ, 148
$\mathcal{R}(\mathcal{F})$	Rees algebra of filtration $\mathcal{F}$, 260
$\mathcal{R}(I)$	Rees algebra of ideal I, 183
$\mathcal{R}(I, M)$	Rees module of ideal I and module M, 296
$\mathcal{R}(I_1, \ldots, I_m)$	multi-Rees algebra of ideals $I_1, \ldots, I_m$, 301
$\mathcal{R}(I_1, \ldots, I_m, M)$	multi-Rees module of ideals $I_1, \ldots, I_m$ and module M, 302
$\widehat{\mathcal{R}}(I)$	extended Rees algebra of ideal I, 186
$\mathcal{R}^{\mathrm{symb}}(I_t)$	symbolic Rees algebra of ideal I_t, 187
$\operatorname{reg} M$	regularity of module M (generalized on p. 276), 39

$\mathrm{reg}_{(0,1)}\, M$	regularity of M with respect to the $(0, 1)$-grading, 288
$\mathrm{reg}_{(1,0)}\, M$	regularity of M with respect to the $(1, 0)$-grading, 288
$\mathrm{relint}(C)$	relative interior of cone C, 175
$\widehat{R^{\mathcal{F}}}$	completion of ring R w.r.t. filtration $\mathcal{F}$, 53
$\widehat{R_{\mathfrak{m}}}$	completion of $R_{\mathfrak{m}}$ w.r.t. ideal $\mathfrak{m}R_{\mathfrak{m}}$, 54
$\rho_N(v)$	supremum of degrees i of nonvanishing components $N_{(i,v)}$, 290
$R^i f_*$	higher direct image functor, 334
(R_k)	Serre condition, 35
$\mathcal{R}^{\mathcal{E}}_{l_i,l_j}$	tautological subquotient sheaf, 331
$R(m, n)$	$K[X]$ for $m \times n$ matrix X, 199
$\mathcal{R}^{\mathcal{E}}_{n-l}$	tautological rank $(n - l)$ subsheaf, 330
$R \# S$	Segre product of graded algebras R and S, 107
$\mathrm{RSK}(\Sigma)$	monomial (or 2-line array) obtained from Σ by RSK, 115
$\mathrm{RSK}(f)$	value of f under linear map RSK, 116
$\mathrm{RSK}(I)$	image of ideal I under linear map RSK, 122
$R^{(i,*)}$	ith homogeneous component of R w.r.t. $(1, 0)$-grading, 288
$R^{(*,j)}$	jth homogeneous component of R w.r.t. $(0, 1)$-grading, 288
$R^{(*,0)}$	subring $\bigoplus_i R_{(i,0)}$ of bigraded ring, 287
$R^{(0,*)}$	subring $\bigoplus_j R_{(0,j)}$ of bigraded ring, 287
$\mathrm{sat}(I)$	saturation of ideal I, 299
$\mathfrak{S}_d$	symmetric group
sign	sign of permutation
`shape`	recursive function to generate shapes in $\mathcal{Z}(\Sigma)$, 390
$\sigma \preceq \tau$	σ precedes τ in dominance order, 78
$\sigma(c)$	truncation of the shape σ, 382
$\Sigma_d(V)$	submodule of symmetric power, 326
σ_G	coinverse on $K[G]$, 351
Σ^{sat}	saturation of a set of shapes in $\mathcal{P}(d)$, 393
$\Sigma_{\sigma,l}$	set of rectangular shapes associated to (σ, l), 457
$(\sigma\lvert\tau)$	bi-shape, 201
(S_k)	Serre condition, 35
$\mathrm{SL}(r, K)$	group of $r \times r$ matrices of determinant 1, 220
$\underline{\mathrm{Spec}}$	relative Spec, 329
$\sqrt{I}$	radical of I
$\mathrm{sr}(\psi)$	small rank of ψ, 209
$\mathfrak{succ}^{\leq}(\sigma, l)$	another notation for $\mathfrak{succ}(\sigma, l)$, 453
$\mathfrak{succ}(\sigma, l)$	l-successors of the shape σ with respect to $\leq$, 380
$\mathfrak{succ}^{\subset}(\sigma, l)$	l-successors of σ with respect to $\subset$, 453
$\mathrm{supp}(f)$	set of monomials with nonzero coefficient in f, 2
$\mathrm{supp}_{\mathbb{G}}(H)$	set of shapes in $\mathbb{G}$-module H, 213

$\mathbb{S}_{\underline{v}}(-)$	Schur functor associated to the weight $\underline{v}$, 362
$\mathrm{Sym}^\bullet(\mathcal{E})$	symmetric algebra of locally free sheaf, 329
$\mathrm{Sym}^\bullet(V)$	symmetric algebra of free module, 326
$\mathrm{Sym}^d\,\mathcal{E}$	symmetric power of locally free sheaf, 328
$\mathrm{Sym}^d\,V$	symmetric power of free module, 325
$t_0(M)$	minimum degree needed to generate module M, 277
$T^d\mathcal{E}$	tensor power of locally free sheaf, 327
$(T^dV)_{\mathfrak{S}_d}$	coinvariants for the symmetric group action on tensors, 325
T^dV	tensor power of free module, 325
$\Theta_d(V)$	submodule of tensor power, 326
$t_i(M)$	largest degree in ith Koszul homology of M, 279
$\mathbf{T}_n$	algebraic group of upper triangular matrices (group scheme), 354
Tr	distinguished map $F_*R \to R$, 236
$U(I)$	open subset of spectrum defined by ideal I, 50
$\mathbf{U}_n$	unipotent group (group scheme), 354
$U^+(n,K)$	upper unipotent $n \times n$ matrices, 97
$U^-(n,K)$	lower unipotent $n \times n$ matrices, 97
$\mathbb{U}$	$U^-(m,K) \times U^+(n,K)$, 98
$V^\vee$	dual of a free module, 327
$V(I)$	closed subset of spectrum defined by ideal I, 50
$v_\mathfrak{p}$	valuation defined by prime ideal $\mathfrak{p}$, 88
$\underline{v}(r,s;\alpha,\beta)$	weight used to describe syzygies of determinantal ideals, 471
$\mathcal{V}^{(\sigma)}, \mathcal{V}^{(\Sigma)}$	sheaf defined by shape, 393
$\mathcal{W}$	Weyl algebra, 468
$X(G)$	character group of G, 353
$X_t, X_t(m,n)$	Zariski closure of Y_t, 208
$Y_t, Y_t(m,n)$	set of exterior powers of linear maps, 208
$\mathcal{Z}(I^{(\Sigma)}),\ \mathcal{Z}(\Sigma)$	combinatorial set indexing a natural filtration on $S/I^{(\Sigma)}$, 382
$\mathcal{Z}(I_\Sigma),\ \mathcal{Z}^{\subset}(\Sigma)$	combinatorial set indexing a natural filtration on S/I_Σ, 454
$\mathbb{Z}^n_{\mathrm{dom}}$	set of dominant weights, 341
$\mathcal{Z}^{(d)}_p$	special notation for the set $\mathcal{Z}(I^{(d)}_p)$, 391
$\underline{z}(r)$	modification of the weight $\underline{z}$, 436
$\mathcal{Z}^{\leq}(\Sigma)$	another notation for $\mathcal{Z}(I^{(\Sigma)})$, 454